Experimental Modal Analysis and Computational Model Updating of a Car Body in White

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Abstract

To validate Finite Element models, test data, e.g. from an experimental modal analysis, may be utilized. The modal data themselves must be highly accurate since they form the basis for all subsequent validation efforts. This implies that systematic test design accompanied by careful modal testing is mandatory.

If the deviations between test and analysis are not acceptable (poor test/analysis correlation), the idealization of the investigated elastomechanical system must be reviewed. Because of the number of uncertain model parameters usually being very high for industrial applications, an appropriate 'manual' update based on engineering skills will most likely fail. Here, computational model updating techniques must be applied, which allow for a simultaneous update of multiple model parameters.

In this paper an integrated validation strategy is presented that takes into account the complete process chain from model based test design, over modal testing, data evaluation, test/analysis correlation to computational model updating. By means of a real car body in white the single steps of the validation strategy will be highlighted, and it will be shown that very encouraging validation results can be obtained even for very complex systems.

1 Introduction

The fidelity of structural mechanical Finite Element analyses (FEA) can be evaluated by using data from static or dynamic tests. Especially, eigenfrequencies and eigenvectors are employed, which can be identified from vibration tests by means of experimental modal analysis (EMA) ([1], [2]). The deviations between test and analysis allow for an evaluation of the quality of the utilized Finite Element model. If the deviations between test and analysis are not acceptable, the idealization of the investigated elastomechanical system is to be reviewed and eventually updated for receiving a validated Finite Element model. In order to keep uncertainties from the experimental investigations as small as possible and for generating an optimal data base for the following model validation thorough test planning and test execution is vital.

If the structure of the Finite Element model with respect to discretization, chosen element types etc. is correct (see for instance [3]), the test/analysis deviations can already be minimized by changing appropriate parameters based on the experience of the engineer in charge. However, proceeding this way bears limits for real elastomechanical systems due to the large number of parameters to be considered. Here, methods for computational model updating (CMU) are to be applied, which allow for a

simultaneous update of multiple model parameters (see for instance [4], [5]). These techniques minimize the test/analysis deviations and enable the validation of the Finite Element model.

In practical applications the structure of Finite Element models is often not correct. However, the techniques for computational model updating can be adopted as well, but the final parameter changes often do not allow for a physical interpretation. They are mathematical substitutes for solely reducing the deviations between test and analysis. If the individual parameter changes are acceptable or if the Finite Element model is to be revised often depends on the field of application of the particular Finite Element model. A large and physically not interpretable increase of the shell thickness might be irrelevant for vibration analyses but might be unacceptable for stress analyses.

For the project presented in this paper the test planning and the computational model updating is supported by **ICS.sysval** a special MATLAB® based software package for model validation, developed by **ICS** and Professor Michael Link of the University of Kassel. Among other things, this software tool allows for the direct update of large scale MSC.NastranTM Finite Element models onto experimental modal data (eigenfrequencies and mode shapes). Therefore, especially the solvers for eigenvalue and eigenvector sensitivity analysis under 'Solution 200' (optimization) are utilized.

2 Theory Overview

The foundation for updating physical stiffness, mass and damping parameters is a parameterization of the system matrices according to equations (1) (see also [4], [5]):

$\mathbf{K} = \mathbf{K}_{\mathrm{A}} + \sum \alpha_{\mathrm{i}} \mathbf{K}_{\mathrm{i}}$	$i = 1 \dots n_{\alpha}$	(1a)
$\mathbf{M} = \mathbf{M}_{\mathrm{A}} + \sum \beta_{\mathrm{j}} \mathbf{M}_{\mathrm{j}}$	$j = 1 \ \ n_\beta$	(1b)
$\mathbf{D} = \mathbf{D}_{\mathrm{A}} + \sum \gamma_{\mathrm{k}} \mathbf{D}_{\mathrm{k}}$	$k{=}1\ldotsn_\gamma$	(1c)
with: $\mathbf{K}_{A}, \mathbf{M}_{A}, \mathbf{D}_{A}$	initial analytical stiffness, mass and damping matrices	

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$\boldsymbol{p} = [\alpha_i \; \beta_j \; \gamma_k]$	vector of unknown design parameters
$\mathbf{K}_{i}, \mathbf{M}_{j}, \mathbf{D}_{k}$	given substructure matrices defining location and type of model uncertainties

This parameterization permits the local adjustment of uncertain model regions. By utilizing equations (1) and appropriate residuals, which consider different test/analysis deviations, the following objective function can be derived:

$$\mathbf{J}(\mathbf{p}) = \Delta \mathbf{z}^{\mathrm{T}} \mathbf{W} \Delta \mathbf{z} + \mathbf{p}^{\mathrm{T}} \mathbf{W}_{\mathrm{p}} \mathbf{p} \rightarrow \min$$

with:	$\Delta \mathbf{z}$	residual vector
	$\mathbf{W}, \mathbf{W}_{p}$	weighting matrices

The minimization of the objective function (2) yields the desired design parameters **p**. The second term on the right hand side of equation (2) is used for constraining the parameter variation. The weighting matrix must be carefully selected, as for $W_p >> 0$ no parameter changes will occur (see also [4]).

The residuals $\Delta \mathbf{z} = \mathbf{z}_{T} - \mathbf{z}(\mathbf{p})$ (\mathbf{z}_{T} : test data vector, $\mathbf{z}(\mathbf{p})$: corresponding analytical data vector) are usually nonlinear functions of the design parameters. Thus, the minimization problem is also nonlinear and is to be solved iteratively. One solution is the application of the classical sensitivity approach (see [5]). Here,

(2)

the analytical data vector is linearized at point 0 by means of a Taylor series expansion truncated after the linear term. Proceeding this way leads to:

$$\Delta \mathbf{z} = \Delta \mathbf{z}_0 - \mathbf{G}_0 \,\Delta \mathbf{p} \tag{3}$$

with: $\Delta \mathbf{p} = \mathbf{p} - \mathbf{p}_0$	design parameter changes
$\Delta \mathbf{z}_0 = \mathbf{z}_{\mathrm{T}} - \mathbf{z}(\mathbf{p}_0)$	test/analysis deviations at linearization point 0
$\mathbf{G}_0 = \partial \mathbf{z} / \partial \mathbf{p} _{\mathbf{p}=\mathbf{p}_0}$	sensitivity matrix at linearization point 0
\mathbf{p}_0	design parameters at linearization point 0

As long as the design parameters are not bounded the minimization problem (2) yields the linear problem (4). The latter is to be solved in each iteration step for the current linearization point:

$$(\mathbf{G}_0^{\mathrm{T}} \mathbf{W} \, \mathbf{G}_0 + \mathbf{W}_{\mathrm{p}}) \,\Delta \mathbf{p} = \mathbf{G}_0^{\mathrm{T}} \mathbf{W} \,\Delta \mathbf{z}_0 \tag{4}$$

For $\mathbf{W}_{p} = \mathbf{0}$ equation (4) represents a standard weighted least squares approach. Of course, any other mathematical minimization technique can be applied for solving equation (2).

In contrast to the assembly of the analytic stiffness and mass matrix, the generation of the analytic damping matrix is usually a difficult task. For treating system damping in an update process modal damping parameters can be utilized alternatively. For further discussions on this topic it is referred to the literature (see for instance [4], [8]).

Commonly, the eigenvalue and the eigenvector residuals are employed. Here, the analytical eigenvalues (squares of the eigenfrequencies) and eigenvectors are subtracted from the corresponding experimental results. The residual vector in this case becomes:

$$\Delta \mathbf{z}_{0} = \begin{bmatrix} \boldsymbol{\lambda}_{\mathrm{T}i} - \boldsymbol{\lambda}_{i} \\ \mathbf{x}_{\mathrm{T}i} - \mathbf{x}_{i} \end{bmatrix}_{0} , i = 1, ..., n$$
(5)

with:	λ_{Ti}, λ_i	test/analysis vectors of eigenvalues
	$\mathbf{X}_{\mathrm{Ti}}, \mathbf{X}_{\mathrm{i}}$	test/analysis mode shape vectors

The correlation between analytical data and test data is accomplished by means of the MAC value of the eigenvectors:

$$MAC \coloneqq \frac{\left(\mathbf{x}_{T}^{\mathrm{T}} \mathbf{x}\right)^{2}}{\left(\mathbf{x}_{T}^{\mathrm{T}} \mathbf{x}_{T}\right)\left(\mathbf{x}^{\mathrm{T}} \mathbf{x}\right)}$$
(6)

which states the linear dependency of two vectors \mathbf{x}_T , \mathbf{x} . A MAC value of one denotes that two vectors are collinear and a MAC value of zero indicates that two vectors are orthogonal.

The sensitivity matrix for the residual vector introduced in equation (5) is given by equation (7). The calculation of the partial derivatives can be found for instance in references [4] and [5].

$$\mathbf{G}_{0} = \begin{bmatrix} \frac{\partial \boldsymbol{\lambda}_{i}}{\partial \mathbf{p}} \\ \frac{\partial \mathbf{x}_{i}}{\partial \mathbf{p}} \end{bmatrix}_{0} , i = 1, ..., n$$

If real eigenvalues and eigenvectors are employed, the adjustment of damping parameters is not possible. The corresponding sensitivities equal zero since the real eigenvalues and eigenvectors depend solely on the stiffness and the mass parameters of the system.

3 Model Validation Strategy

The model validation is accomplished through computational model updating of physical parameters (stiffness and inertia parameters) of the Finite Element model by minimizing the deviations between the identified and the analytical eigenvalues and mode shapes. It is presumed that all deviations between test and analysis are exclusively based on uncertainties of the Finite Element model. For keeping the inevitable uncertainties from the test side as small as possible and for creating a reliable data base for the following validation tasks thorough test planning and test execution is an integral part of the validation strategy. The principle proceeding is depicted in Figure 1.



Figure 1: Model validation strategy

The test planning utilizes the given Finite Element model, which enables not only the test design but also considerably simplifies the later correlation with the analytical results (Finite Element model and test model 'match'). The test planning should cover the following aspects:

- selection of relevant target modes
- selection of measurement degrees of freedom with respect to
 - essential test information
 - sufficient spatial resolution of the target modes

- coincidence of measurement and Finite Element nodes
- accessibility of the measurement nodes
- redundancy of the measurement degrees of freedom
- robustness of the test model
- selection of exciter positions (if possible, simultaneous excitation of all target modes)
- sufficient frequency resolution (for following identification methods)

Test planning and computational model updating is conducted by a special MATLAB® based software package (**ICS.sysval**, [6]), which is developed by **ICS** and Professor Michael Link of the University of Kassel. This software tool takes advantage of the analysis capabilities of MSC.NastranTM, particularly the sensitivity module within 'Solution 200' (optimization), which enables the handling of large scale Finite Element models. The necessary parameter changes are directly applied to the so called 'bulk data' section of the MSC.NastranTM input file. Typical parameters are for instance shell thicknesses, beam section properties, Young's moduli, and densities. However, virtually all physical parameters, which can be considered in an eigenvalue and eigenvector sensitivity analysis by MSC.NastranTM, can be used for model updating.

After successfully updating the stiffness and inertia properties (physical parameters) modal damping parameters (modal parameters) can be adjusted subsequently by minimizing the deviations in the resonance regions between measured and simulated frequency response functions. However, this topic is not covered in this paper and it is referred to reference [8].

A common difficulty in computational model updating is the proper choice of appropriate model parameters. Besides selection with engineering experience automated methods may be applied [7], which currently do not deliver a reliable prediction. Another possibility to select parameters is provided by a sensitivity analysis. Here the sensitivity matrix according to equation (7) is computed for a set of suitable parameters. In a subsequent investigation those parameters are identified, which have a significant influence on the analysis results. However, the sensitivity analysis does not supply any information on the physical relevance of a particular parameter but detects merely its potential to change the analysis results.

4 Example: Car Body in White

The application of the presented validation strategy will be demonstrated by means of the car body in white depicted in Figure 2, which is currently investigated in the frame of the cooperation 'work group 6.1.19 structure optimization and acoustics' of the German car industry.

The Finite Element model consists of about:

- 142.000 nodes
- 130.000 elements.
- 3.500 spot weld elements

The spot welds are modeled by means of MSC.NastranTM 'CWELD' elements. These elements are currently already used by some automotive companies.

Goal of the model validation is to correctly predict the structural dynamics of the body in white up to a frequency of about 100 Hz.



Figure 2: Finite Element model of the body in white

4.1 Test Planning

As already mentioned in section 3, the test planning provides an important basis for all subsequent investigations. It is especially guaranteed that all required information is collected during the tests, which is necessary for the following validation steps. For the analyzed body in white the following planning steps are performed:

a) Selection of target mode shapes

First, the boundary conditions, the frequency range, and the relevant mode shapes are to be determined. The body in white will be investigated in a free/free configuration which can be relatively easily realized in the test setup by means of bungee cords or air springs. Since the goal of the model validation is a prediction of the structural dynamics of the body in white up to a frequency of about 100 Hz all analytic mode shapes in this frequency range are to be considered in the test planning.

b) Selection of measurement degrees of freedom

Since a reliable orientation of the accelerometers on the body in white is difficult due to the curvature of the car body, only measurement degrees of freedom normal to the sheet surfaces are considered in the test design. This provides in addition the possibility to perform a roving hammer data acquisition. Of course, the selection of the measurement degrees of freedom should also ensure the unique classification of the individual mode shapes (keyword: spatial aliasing).

Moreover, all aspects mentioned in section 3 like the essential test information (for instance determined by automatic methods, see also [9]), the sufficient spatial resolution of the target modes (goal: preferably diagonal shape of the auto MAC matrix), the coincidence of measurement and Finite element nodes (vital for correlation), the redundancy of the measurement degrees of freedom (for uncertainties in some measurement degrees of freedom), and the robustness of the test model (due to uncertainties in the Finite Element model used for test planning) should be taken into account.

The final test model resulting from test planning is shown in Figure 3 and the corresponding auto MAC matrix of the analytical mode shapes at the measurement degrees of freedom is depicted in Figure 4. The overall spatial resolution is sufficient. Only two mode shapes (No. 24 and 27 as well as No. 32 and 35) exhibit off diagonal couplings larger than 60 %. These are relatively local mode shapes, which require a considerably higher resolution. However, for keeping the measurement efforts manageable a higher resolution of the measurement mesh is not applied.



Figure 3: Test model of the body in white with measurement degrees of freedom



Figure 4: Auto MAC matrix of analytical mode shapes at measurement degrees of freedom

c) Selection of exciter degrees of freedom

The identification of the exciter degrees of freedom is done in two steps. First, a preliminary subset of appropriate exciter degrees of freedom is calculated using a special automatic method (see reference [9]). Next, the final exciter degrees of freedom are defined by means of 'mode indicator values'. For a given exciter position a mode indicator value of zero at a certain eigenfrequency states that the corresponding mode shape can be fully excited (satisfaction of the so called phase resonance criteria). In contrast, a value of one means that the corresponding mode shape cannot be excited at all at the chosen exciter position.

The strategy is to select several exciter positions in such a way that each mode shape can be sufficiently excited at least at one exciter position. The accessibility of the exciter position for a modal exciter test is also taken into account. Figure 5 shows the mode indicator values for the first 20 target modes at the four selected references according to Figure 6. Obviously, each mode shape can be excited at least by one of the defined exciter positions.



Figure 5: Mode indicator values of the body in white



Figure 6: Test model of the body in white with exciter degrees of freedom

d) Selection of frequency resolution

The applied techniques to identify the experimental modal data require a sufficient frequency resolution, especially in the lower frequency range. Therefore, a minimal frequency resolution based on the first elastic eigenfrequency and the expected modal damping is determined. Since the real damping behavior is usually not known in advance, these results are to be verified before the final test execution.

4.2 Test setup and Preliminary Investigations

For approximating the free/free boundary conditions of the Finite Element model the car body is mounted on four air springs as depicted in Figure 7. If necessary, the suspension can be considered in the Finite Element model by means of springs. The properties of the springs can be estimated from the experimental rigid body modes.

Different preliminary studies with impact hammer and modal exciters are performed to investigate the real behavior of the body in white. Especially, the linearity is reviewed by means of modal exciter tests with different levels of excitation as well as by coherence and reciprocity examinations. All together, the investigated car body shows a sufficiently linear behavior.



Figure 7: Mounting of the body in white using air springs

As the test results with the impact hammer exhibit a very good consistency with the results of the modal exciter tests, the final test will be executed as a roving hammer test with fixed accelerometers (references). Proceeding this way also avoids the so called 'mass loading', caused by moving accelerometer masses. 'Mass loading' effects easily arise at large surface areas and can generate frequency shifts of the resonance peaks. In regions with large modal density these effects cannot be compensated by common identification methods and thus produce unreliable test results.

Since the measurement degrees of freedom are manually mapped onto the car body, an uncertainty regarding the real position of the measurement points exists. For quantifying possible deviations a digitization of the measurement points using super sonic triangulation is applied. The acquired real measurement point positions prove good accordance with the Finite Element model.

4.3 Experimental Modal Analysis

The analysis of the acquired test data is conducted by two completely different identification algorithms, one working in the time domain (Polyreference) and one working in the frequency domain (Direct Parameter Estimation). The identified sets of modal data are subsequently correlated allowing for an assessment of the individual quality of the measured eigenfrequencies, mode shapes etc. This is of particular importance for the subsequent model validation, since only experimental results with a sufficient quality should be utilized.

4.4 Initial Correlation

For evaluating the model quality the frequency deviations between test and analysis as well as the MAC values of the corresponding mode shapes according to equation (6) are employed. Besides the correlation table the MAC matrix will also be posted for assessing the quality of the correlation.

The initial correlation for the investigated body in white is given by Table 1 and Figure 8, considering only mode shapes with a frequency deviation less than 30 % and a MAC value larger than 50 %. Usually, a tolerance limit of 70 % is acceptable to ensure a consistent correlation of the mode shapes used for the updating. However, in this case a decrease is necessary for considering all relevant mode shapes in the initial correlation and especially in the following computational model updating.

#	EMA ¹⁾	FEA	Δf [%]	MAC [%]	#	EMA ¹⁾	FEA	∆f [%]	MAC [%]
1	1	7	-7,04	98,24	8	12	18	-3,07	66,41
2	2	8	0,19	87,42	9	13	19	-0,02	72,49
3	3	9	-6,17	63,55	10	14	20	-1,85	86,89
4	5	13	0,83	73,20	11	15	22	1,39	78,18
5	6	11	-11,72	61,13	12	17	24	-0,07	67,88
6	8	14	-3,39	79,49	13	18	26	-1,07	96,49
7	10	15	-6,55	66,51	14	19	27	-1,49	76,60

¹⁾ without rigid body modes

Table 1: Initial correlation

Obviously, even by reducing the tolerance limit to 50 % not all mode shapes can be correlated ('gaps' on the main diagonal of the MAC matrix). Moreover, some couples show frequency deviations larger then 10 %.



Figure 8: MAC matrix, initial correlation

4.5 Sensitivity Analysis

For reducing the multitude of potential parameters to a subset of parameters, which have a significant influence on the model behavior, a sensitivity analysis is performed by computing the eigenvalue and eigenvector sensitivities for all potential parameters. Subsequently, the most promising parameters for each mode shape are determined using the sensitivity module of **ICS.sysval**. These parameters constitute the basis for the following computational model updating.

4.6 Remodeling und Computational Model Updating

In the investigated example a division of the updating task into two individual steps proves to be very efficient. In both cases Young's moduli are used for the updating (for preserving the total mass of the car body).

In the first step areas with large, physically not interpretable, parameter changes are identified by applying computational model updating methods. These areas are then remodeled using the CAD data of the geometry, which already leads to an improvement of the model quality. The results after remodeling are collected in Table 2 and Figure 9, respectively.

The remodeling already yields a significant increase of the model quality. On the one hand the MAC values, especially in the lower frequency range, can be increased over 90 % and on the other hand the frequency deviations are noticeably reduced. However, the correlation in the range of the sixth and seventh as well as the ninth and tenth measured mode is still not fully satisfactory.

#	EMA ¹⁾	FEA	<mark>Δf</mark> [%]	MAC [%]	#	EMA ¹⁾	FEA	<mark>Δf</mark> [%]	MAC [%]
1	1	7	-4,51	98,62	10	11	17	-0,14	68,37
2	2	8	3,01	97,51	11	12	18	-0,57	88,51
3	3	9	-1,49	96,22	12	13	19	0,19	72,25
4	4	10	-0,16	90,12	13	14	20	-1,31	89,43
5	5	11	1,26	90,92	14	15	21	0,83	65,52
6	6	12	0,65	68,55	15	16	22	0,18	66,29
7	7	13	-0,71	59,68	16	17	24	0,48	86,01
8	8	15	0,22	94,88	17	18	25	-0,98	98,19
9	10	14	-6,35	52,61					

¹⁾ without rigid body modes

Table 2: Correlation after remodeling



Figure 9: MAC matrix, after remodeling

For further increasing the model quality multiple updating runs are performed in a second step utilizing the remodeled Finite Element model. The final results are summarized in Table 3 and Figure 10.

Comparing these results with Table 2 and Figure 9 shows an additional increase of the model quality achieved by the subsequent computational model updating. Both, the mentioned 'gaps' on the main diagonal of the MAC matrix can be filled and the frequency deviations can be further reduced. Besides the

#	EMA ¹⁾	FEA	<mark>∆f</mark> [%]	MAC [%]	#	EMA ¹⁾	FEA	<mark>∆f</mark> [%]	MAC [%]
1	1	7	-4.35	98.69	10	10	16	0.21	82.22
2	2	8	0.75	97.54	11	11	17	0.09	69.57
3	3	9	-1.49	95.49	12	12	18	-0.90	85.80
4	4	10	1.10	94.38	13	13	19	0.15	75.85
5	5	11	-0.50	93.86	14	14	20	-1.35	92.13
6	6	12	-1.83	95.05	15	15	22	1.49	63.63
7	7	13	-2.67	90.49	16	17	24	0.85	85.47
8	8	14	-1.26	95.39	17	18	25	-0.71	98.09
9	9	15	-1.55	80.52					

first eigenfrequency all frequency deviations are now below 3 %. However, it is to mention that the total deviation for the first eigenfrequency is less than 1 Hz.

¹⁾ without rigid body modes





Figure 10: MAC matrix, after computational model updating

5 Summary

This paper presents the validation of the Finite Element model of a body in white using a special software package for computational model updating, which enables the direct update of large scale MSC.NastranTM Finite Element models.

For keeping the inevitable uncertainties from the test side as small as possible a thorough test planning is essential. By utilizing the given Finite Element model both measurement degrees of freedom and exciter positions are first virtually defined and then transferred onto the body in white. For avoiding so called 'mass loading' effects the data acquisition is performed using a roving hammer excitation with fixed accelerometer positions.

Besides the direct increase of the model quality, the computational model updating provides the additional opportunity to identify regions, where remodeling already yields an improvement of the model quality. The test/analysis correlation before and after the computational model updating as well as the remodeling exhibits a noticeable reduction of the frequency deviations along with an increase of the MAC values over a broad frequency range.

These investigations are currently expanded towards attachment parts like doors as well as subassemblies in the frame of the cooperation 'work group 6.1.19 structure optimization and acoustics' of the German car industry. A first goal is to sufficiently represent the structural dynamics for having thorough basis for subsequent investigations on acoustic phenomena.

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This paper is dedicated to Mr. Jörn Frappier, who died in a tragic accident at the beginning of this year.

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