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ON THE IDENTIFICATION OF RIGID BODY PROPERTIES OF AN ELASTIC SYSTEM

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ABSTRACT. *This paper presents a procedure for identification of the full set of rigid body properties (overall mass, center of gravity location and moments of inertia) of a given elastomechanical system which is based on experimental modal analysis. The approach uses measured inertance frequency response functions (FRF) up to the first elastic natural frequency of the free/free system which are obtained either by testing the system suspended in soft springs or by testing the fixed/free system with an additional measurement of the interface forces. In a first step the underlying rigid body response is extracted from the FRFs by taking the influences of the elastic modes into account. A second step then uses these data to estimate the rigid body properties.*

The theoretical foundation of the identification procedure is presented as well as the requirements for setting up an appropriate vibration test. Furthermore the procedure will be classified with respect to existing methods and advantages and disadvantages are compared.

A fan/motor unit which is used in air conditioning systems of cars has been tested and the identification procedure was applied in order to estimate the rigid body properties. The test set-up will be described and the identification results will be presented. In addition a comparison to results coming from standard pendulum testing will show a very good correlation and thus emphasizes the efficiency of the method.

Keywords: *rigid body properties, identification*

NOMENCLATURE

ξ^S, η^S, ζ^S	location of center of gravity w.r.t. reference point
$\Theta^A_{\xi\xi}$, ...	moments of inertia w.r.t. reference point
$\ddot{\xi}^A$, ...	translational accelerations of point A
$\ddot{\alpha}^A$, ...	circular accelerations about point A
f^A_ξ, ...	forces at point A
t^A_ξ, ...	moments at point A
$\mathbf{a}$, $\mathbf{A}$	vectors of accelerations, time/frequency domain
$\mathbf{B}$, Ω	measurement matrices
Δ	indictor value
$\mathbf{f}$, $\mathbf{F}$	vectors of forces/moments, time/frequency domain
$\mathbf{H}$	frequency response function (FRF) matrix
m	overall mass
$\mathbf{M}^A$	rigid body mass matrix w.r.t. point A
σ, $\mathbf{c}$	estimation vectors
$\mathbf{u}$	vector of displacements
$\mathbf{W}$	weighting matrix
$\mathbf{X}_R$	rigid body modes

1 INTRODUCTION

At the present time various methods are available in order to identify the rigid body properties of a given system. These methods may be divided into two main categories:

- time domain methods
- frequency domain methods

The first *time domain methods* to mention are the classical static methods and pendulum methods which are still commonly used [HOLZWEISSIG]. They provide reliable results within a very short testing time, if performed accurately. Of course it would be desirable to avoid these additional tests by extracting the rigid body properties from vibration tests if they are executed anyway.

Other time domain methods are based on the evaluation of vibration test data. Pandit, et. al [PANDIT] for instance focus on the time domain equation of motion of a rigid body under elastic, damped mounting conditions. Hahn et. al [HAHN] use the time domain test data of a six-axes shaking table system where the exciter forces are measured in addition to the acceleration responses.

Advantage of the time domain methods is the direct evaluation of the test data without the necessity of a transformation into frequency domain and its inherent signal processing problems. The identification algorithm can directly be applied to the test data. A disadvantage is, however, that if the system under observation does not behave as a rigid body in the excited frequency range low pass filtering of the test data must be performed. This requires an additional analysis of the frequency content of the test data. Furthermore the influence of the structure's elastic response in the used frequency range may not be eliminated by low pass filtering alone if the first elastic natural frequency of the system is very low. In this case time domain methods cannot be applied at all unless special account is taken for such effects.

Frequency domain methods on the other hand bear the possibility to circumvent this disadvantage of the time domain methods because a separation of rigid and elastic system behavior is possible even if the first elastic natural frequency is very low. The frequency domain methods may be subdivided into three categories:

- modal parameter methods
- methods of direct physical parameter identification
- residual inertia methods (massline methods)

The modal parameter methods are based on the orthogonality relation between the mass matrix of the system and the rigid body modes (see e.g. [BRETL]). Advantage of these methods is that they use the results of a preceding experimental modal analysis of the quasi free/free system (suspended in soft springs). The experimental modal analysis itself may be performed using various well established methods (see e.g. [EWINS, NATKE]). A disadvantage is that in general not all rigid body modes may be excited in a real test.

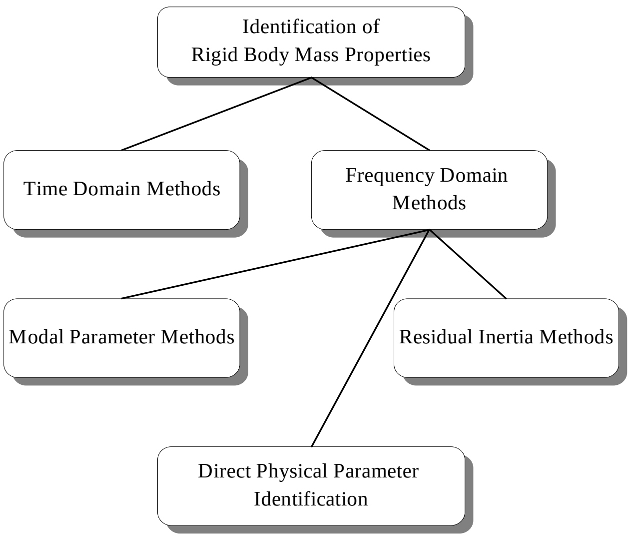
The methods of direct physical parameter identification focus on a fit of system matrices to identified frequency response functions. General methods take into account the elastic behavior of the system [LINK-1, LINK-2] which does not restrict them to ideal rigid systems. A disadvantage is that, as for the modal parameter methods, in general not all rigid body modes of the quasi free/free system may be excited in a real test.

Special methods of direct physical parameter identification are based on a fit of the equation of motion of a rigid body under elastic, damped mounting conditions to identified frequency response functions ([MANGUS-1, NAKAMURA]). A new approach [MANGUS-2] proposes the measurement of the vibration response of a fixed/free system under base excitation with an additional measurement of the interface forces. Advantage of the special methods is their simple formulation which is better suited for the given identification problem. A disadvantage is again the requirement that the system's response must be governed by the rigid body behavior. However in the frequency domain a separation of the rigid and the elastic behavior is possible and merely increases the analysis effort.

The residual inertia methods (massline methods) which have been under investigation in many publications recently may be regarded as a special case of the special methods of direct physical parameter identification. Basis for these methods is the equation of motion of a rigid body under free/free boundary conditions with respect to a given reference point. Input for these methods are residual inertiae which can be extracted from vibration test data in various ways [SCHEDLINSKI-2].

Bretl and Conti [BRETL] have been one of the first to publish a residual inertia method. They extract the residuals directly from frequency response functions of the system in a low frequency suspension using the frequency range between the highest rigid body mode and the first elastic mode (massline). Wei and Reis [WEI] identify the residual inertiae with a special curve fitting procedure. Here the residual inertiae are a by-product of the modal identification of the first elastic mode. Okuzumi [OKUZUMI] proposes an iterative method in order to identify the rigid body properties with respect to the center of gravity which has the advantage, that the identified rigid body properties do not have to be transformed to the center of gravity in a subsequent step. A thorough investigation of the identification equation itself can be found in [FREGOLENT]. In [SCHEDLINSKI-1] the authors show that the additional measurement of interface forces during shaking table testing of the fixed/free system provide the information to estimate the free/free system's FRFs. These FRFs can subsequently be used to estimate the rigid body properties of the tested system [SCHEDLINSKI-2] by applying a residual inertia method.

Figure 1: Overview of identification methods



In general the frequency domain methods seem to have the highest development level in the literature. Toivola and Nuutila [TOIVOLA, NUUTILA] have performed an analytical and an experimental study where they compared a modal parameter method, a method of direct physical parameter identification and a residual inertia method. The residual inertia method provided the most accurate results the modal parameter method the second best and the method of direct physical parameter identification the poorest. Figure 1 above gives an overview of the discussed methods.

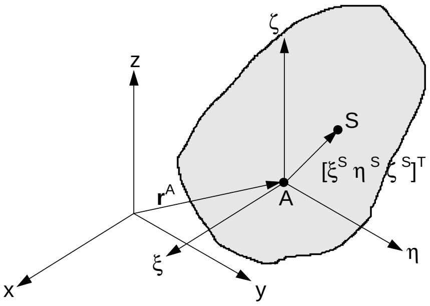
This paper here shows how to estimate rigid body properties from free/free FRF data using a residual inertia method. The approach explicitly takes the elastic influences into account which makes it appropriate for test data of systems which do not strictly behave as a rigid body in the observed frequency range. An application will be discussed where the needed input data was assembled from testing the system under quasi free/free boundary conditions, i.e. suspended in soft springs. The results will be compared to data coming from weighing and pendulum testing which show an excellent correlation.

2 THEORY

2.1 Equations of motion & transformation of accelerations and forces

The linearized time domain equations of motion of an unrestrained rigid body (see figure 2) written down for an arbitrary reference point A yields equation (1).

Figure 2: Unrestrained rigid body

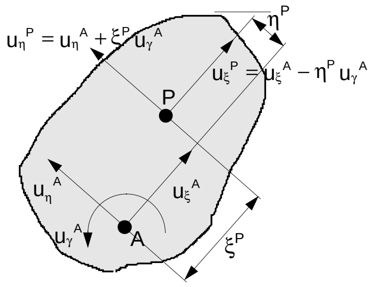


$$\underbrace{\begin{bmatrix} m & 0 & 0 & 0 & m\zeta^S & -m\eta^S \\ 0 & m & 0 & -m\zeta^S & 0 & m\xi^S \\ 0 & 0 & m & m\eta^S & -m\xi^S & 0 \\ 0 & -m\zeta^S & m\eta^S & \Theta^A_{\xi\xi} & -\Theta^A_{\xi\eta} & -\Theta^A_{\xi\zeta} \\ m\zeta^S & 0 & -m\xi^S & -\Theta^A_{\xi\eta} & \Theta^A_{\eta\eta} & -\Theta^A_{\eta\zeta} \\ -m\eta^S & m\xi^S & 0 & -\Theta^A_{\xi\zeta} & -\Theta^A_{\eta\zeta} & \Theta^A_{\zeta\zeta} \end{bmatrix}}_{\mathbf{M}^A} \underbrace{\begin{bmatrix} \ddot{\xi}^A \\ \ddot{\eta}^A \\ \ddot{\zeta}^A \\ \ddot{\alpha}^A \\ \ddot{\beta}^A \\ \ddot{\gamma}^A \end{bmatrix}}_{\mathbf{a}^A} = \underbrace{\begin{bmatrix} f^A_\xi \\ f^A_\eta \\ f^A_\zeta \\ t^A_\xi \\ t^A_\eta \\ t^A_\zeta \end{bmatrix}}_{\mathbf{f}^A} \qquad (1)$$

In general not all translational and rotational accelerations or forces and moments respectively can be measured for the chosen reference point under real test conditions. Therefore translational accelerations and forces are measured at as many accessible locations as desired and subsequently transformed on the reference point. For a rigid body the needed transformation is purely geometric.

Assuming that the displacements $\mathbf{u}^A$ at the reference point A are known the translational displacements $\mathbf{u}^P$ at a given point P can be derived by a simple linear combination (see figure 3 and equation (2)).

Figure 3: Transformation of displacements (2D)



$$\underbrace{\begin{bmatrix} \xi^P \\ \eta^P \\ \zeta^P \end{bmatrix}}_{\mathbf{r}} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & \zeta^P & -\eta^P \\ 0 & 1 & 0 & -\zeta^P & 0 & \xi^P \\ 0 & 0 & 1 & \eta^P & -\xi^P & 0 \end{bmatrix}}_{\mathbf{X}_R^P} \underbrace{\begin{bmatrix} \xi^A \\ \eta^A \\ \zeta^A \\ \alpha^A \\ \beta^A \\ \gamma^A \end{bmatrix}}_{\mathbf{r}^A} \qquad (2)$$

Matrix $\mathbf{X}_R^P$ contains the components of the rigid body modes for point P with respect to the reference point A. Now let $\mathbf{u}^M$ be a vector of displacements at n degrees of freedom. If the corresponding components of the rigid body modes are used to assemble the rows of matrix $\mathbf{X}_R$ we arrive at (3-a) which is an overdetermined system of equations in case of n > 6:

$$\underbrace{\mathbf{u}^M}_{(n,1)} = \underbrace{\mathbf{X}_R}_{(n,6)} \underbrace{\mathbf{u}^A}_{(6,1)} \qquad (3\text{-a})$$

Since $\mathbf{X}_R$ is constant equation (3-a) also holds for the accelerations:

$$\ddot{\mathbf{u}}^M = \mathbf{X}_R \, \ddot{\mathbf{u}}^A \Rightarrow \underbrace{\mathbf{a}^M}_{(n,1)} = \underbrace{\mathbf{X}_R}_{(n,6)} \underbrace{\mathbf{a}^A}_{(6,1)} \qquad (3\text{-b})$$

Equation (3-b) can now be solved with respect to the rigid body accelerations at the reference point in a weighted least squares sense. Therefore a constant diagonal (n,n) weighting matrix $\mathbf{W}$ can be introduced (see equation (4)). If no special ranking of the measured accelerations is intended the unity matrix is taken.

$$\mathbf{a}^A = \left(\mathbf{X}_R^T \, \mathbf{W} \, \mathbf{X}_R\right)^{-1} \mathbf{X}_R^T \, \mathbf{W} \, \mathbf{a}^M \qquad (4)$$

The relation between the force resultants $\mathbf{f}^A$ and the forces applied at n measurement degrees of freedom $\mathbf{f}^M$ is obtained from the equilibrium conditions expressed by the principle of virtual work: The virtual work of the force resultants and their corresponding displacements at point A must be equal to the virtual work of the applied forces and the displacements at the corresponding measurement degrees of freedom:

$$\left(\mathbf{f}^A\right)^T \delta\mathbf{u}^A \overset{!}{=} \left(\mathbf{f}^M\right)^T \delta\mathbf{u}^M$$

Using the relation between the displacements at point A and those at the n measured degrees of freedom (3-a) the following equilibrium equation is obtained:

$$\underbrace{\mathbf{f}^A}_{(6,1)} = \underbrace{\mathbf{X}_R^T}_{(6,n)} \underbrace{\mathbf{f}^M}_{(n,1)} \qquad (5)$$

2.2 Identification equations

Basis for the development of the identification equations are the equations of motion (1). Since only 10 quantities in (1) are unknown the following estimation vector σ can be defined (see e.g. [URGUEIRA]).

$$\sigma = \left[m \quad m\xi^S \quad m\eta^S \quad m\zeta^S \quad \Theta^A_{\xi\xi} \quad \Theta^A_{\eta\eta} \quad \Theta^A_{\zeta\zeta} \quad \Theta^A_{\xi\eta} \quad \Theta^A_{\xi\zeta} \quad \Theta^A_{\eta\zeta} \right]^T \qquad (6)$$

A disadvantage of this estimation vector is that the location of the center of gravity cannot be explicitly estimated so that the error on the estimated overall mass and the error on the coupled terms $m\xi^S$, ... may add. However, an explicit estimation is possible if the overall mass has a priori been determined (e.g. by weighing). Then the number of unknowns is reduced to 9 and the estimation vector does not contain the overall mass (see e.g. [BRETL]). Another possibility is to introduce the estimation vector

$$\sigma = \left[\Delta \quad \xi^S \quad \eta^S \quad \zeta^S \quad \Theta^A_{\xi\xi} \quad \Theta^A_{\eta\eta} \quad \Theta^A_{\zeta\zeta} \quad \Theta^A_{\xi\eta} \quad \Theta^A_{\xi\zeta} \quad \Theta^A_{\eta\zeta} \right]^T \qquad (7)$$

Reassembling (1) then yields:

$$\underbrace{\begin{bmatrix} m\ddot{\xi}^A & 0 & -m\ddot{\gamma}^A & m\ddot{\beta}^A & 0 & 0 & 0 & 0 & 0 & 0 \\ m\ddot{\eta}^A & m\ddot{\gamma}^A & 0 & -m\ddot{\alpha}^A & 0 & 0 & 0 & 0 & 0 & 0 \\ m\ddot{\zeta}^A & -m\ddot{\beta}^A & m\ddot{\alpha}^A & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & m\ddot{\zeta}^A & -m\ddot{\eta}^A & \ddot{\alpha}^A & 0 & 0 & -\ddot{\beta}^A & -\ddot{\gamma}^A & 0 \\ 0 & -m\ddot{\zeta}^A & 0 & m\ddot{\xi}^A & 0 & \ddot{\beta}^A & 0 & -\ddot{\alpha}^A & 0 & -\ddot{\gamma}^A \\ 0 & m\ddot{\eta}^A & -m\ddot{\xi}^A & 0 & 0 & 0 & \ddot{\gamma}^A & 0 & -\ddot{\alpha}^A & -\ddot{\beta}^A \end{bmatrix}}_{\mathbf{B}} \underbrace{\begin{bmatrix} \Delta \\ \xi^S \\ \eta^S \\ \zeta^S \\ \Theta^A_{\xi\xi} \\ \Theta^A_{\eta\eta} \\ \Theta^A_{\zeta\zeta} \\ \Theta^A_{\xi\eta} \\ \Theta^A_{\xi\zeta} \\ \Theta^A_{\eta\zeta} \end{bmatrix}}_{\sigma} = \underbrace{\begin{bmatrix} f^A_\xi \\ f^A_\eta \\ f^A_\zeta \\ t^A_\xi \\ t^A_\eta \\ t^A_\zeta \end{bmatrix}}_{\mathbf{f}^A} \qquad (8)$$

Here the accelerations and the overall mass form the measurement matrix $\mathbf{B}$ and the forces and moments form the force vector $\mathbf{f}^A$. For an ideal estimation Δ must be equal to one. For a non ideal estimation ($\Delta \neq 1$) the deviation from one may be used as an indicator for the quality of the estimation. Equation (8) is now assembled for $i = 1...n_e$ excitation configurations such that the related measurement matrices $\mathbf{B}_{(i)}$ and the force vectors $\mathbf{f}^A_{(i)}$ form the following overdetermined equation system:

$$\underbrace{\begin{bmatrix} \mathbf{B}_{(1)} \\ \mathbf{B}_{(2)} \\ \vdots \\ \mathbf{B}_{(n_e)} \end{bmatrix}}_{\hat{\mathbf{B}}} \sigma = \underbrace{\begin{bmatrix} \mathbf{f}^A_{(1)} \\ \mathbf{f}^A_{(2)} \\ \vdots \\ \mathbf{f}^A_{(n_e)} \end{bmatrix}}_{\hat{\mathbf{F}}} \qquad (9)$$

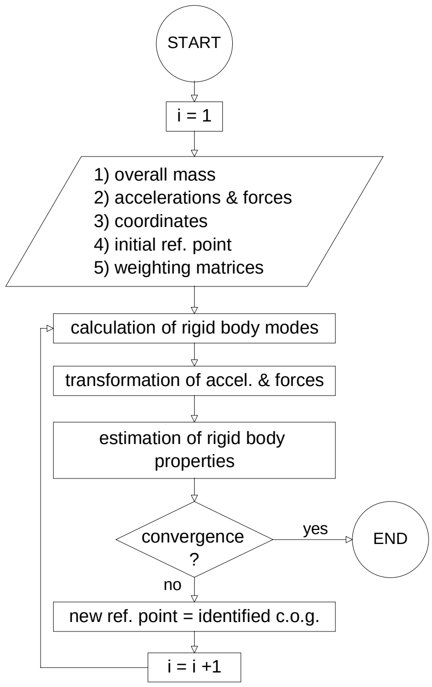
Equation (9) can be solved in a least squares sense only if a minimum of two linear independent excitation configurations have been measured leading to 12 equations for the 10 unknowns. These excitation configurations must be chosen such that all the parameters to be identified can sufficiently be observed.

$$\sigma = \left(\hat{\mathbf{B}}^T \hat{\mathbf{B}}\right)^{-1} \hat{\mathbf{B}}^T \hat{\mathbf{F}} \qquad (10)$$

2.3 The identification algorithm

The identification itself is done by solving equation (9) according to equation (10) non-iteratively for a given reference point A or iteratively. The iterative procedure (figure 4) uses the location of the center of gravity identified in the actual iteration step as the reference point for the next iteration step and thus yields the parameters with respect to the center of gravity after convergence is achieved.

Figure 4: Overview of identification algorithm



Input data for the identification algorithm are estimated rigid body accelerations (see chapter 2.5 below) and the corresponding excitation forces, the coordinates of a (starting) reference point and the coordinates of the measurement locations. The coordinates may reference any global Cartesian coordinate system of choice. Furthermore weighting matrices can be supplied for each excitation configuration to individually emphasize the measured rigid body accelerations.

At first the needed components of the rigid body mode matrix are calculated for the actual reference point. Then the estimated rigid body accelerations and excitation forces are transformed according to equations (4) and (5). After assembling the measurement matrices and the force vectors for all excitation configurations the hyper matrix system (9) is formed and solved according to equation (10). The iteration is performed by repeating the procedure until the identified location of the center of gravity coincides with the actual reference point within a chosen tolerance radius.

2.4 Transformation into frequency domain

Transformation of equation (1) into the frequency domain via:

$$\mathbf{a}^A(t) = \mathbf{A}^A(j\omega)\, e^{j\omega t} \quad , \quad \mathbf{f}^A(t) = \mathbf{F}^A(j\omega)\, e^{j\omega t}$$

with ω - circular frequency, j = $\sqrt{-1}$ yields:

$$\mathbf{M}^A \; \mathbf{A}^A(j\omega) = \mathbf{F}^A(j\omega) \tag{11}$$

It can be seen that (11) is completely equivalent to (1) and thus all the equations derived so far remain valid if frequency domain quantities are used instead of time domain quantities.

2.5 How to determine the rigid body response

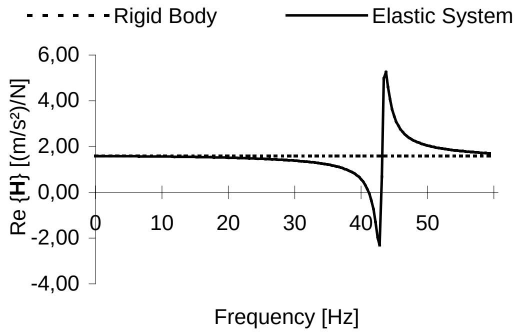
From this point on free/free FRF data will be considered only. Here the force vector to be supplied to the identification algorithm is equal to one at the excitation location and zero elsewhere for the complete frequency range. The accelerations are the system's response at the chosen measurement degrees of freedom (dof) to unit single point force excitation and will be labeled $\mathbf{H}^M(j\omega)$ in this special case.

For an ideal rigid body the FRFs are purely real and represent straight lines if plotted over the frequency range. However for an elastic system the influence of the elastic modes is superimposed over the rigid body response (see figure 5).

For a discrete, linear and time invariant system it can be shown that the real part of $\mathbf{H}^M(j\omega)$ is an even function. Thus the following bi-quadratic approximation is chosen for the real part of each single FRF k which is meaningful up to the first elastic natural frequency of the system:

$$H_k^{M,\,re}(j\omega) = C_0 + C_2\omega^2 + C_4\omega^4 \tag{12}$$

Figure 5: Rigid body vs elastic system



Assembling data at i = 1, ..., n ≥ 3 frequency lines ω_i yields:

$$\underbrace{\begin{bmatrix} H_k^{M,\,re}(j\omega_1) \\ H_k^{M,\,re}(j\omega_2) \\ \vdots \\ H_k^{M,\,re}(j\omega_n) \end{bmatrix}}_{\mathbf{H}_k^{M,\,re}} = \underbrace{\begin{bmatrix} 1 & \omega_1^2 & \omega_1^4 \\ 1 & \omega_2^2 & \omega_2^4 \\ \vdots & \vdots & \vdots \\ 1 & \omega_n^2 & \omega_n^4 \end{bmatrix}}_{\Omega} \underbrace{\begin{bmatrix} C_0 \\ C_2 \\ C_4 \end{bmatrix}}_{\mathbf{c}} \tag{13}$$

Equation (13) can now be solved in a least squares sense:

$$\mathbf{c} = (\Omega^T \Omega)^{-1}\, \Omega^T \; \mathbf{H}_k^{M,\,re} \tag{14}$$

The constant term C_0 in $\mathbf{c}$ now represents an estimation of the underlying rigid body response for measurement dof k. Repeating this procedure for all measurement dof yields the remaining data needed as input for the identification algorithm, i.e. the rigid body acceleration response at the chosen measurement dof.

3 TEST EXAMPLE - FAN/MOTOR UNIT

A fan/motor unit which is used in air conditioning systems of cars (figure 6) has been tested and the identification procedure was applied in order to estimate the rigid body properties.

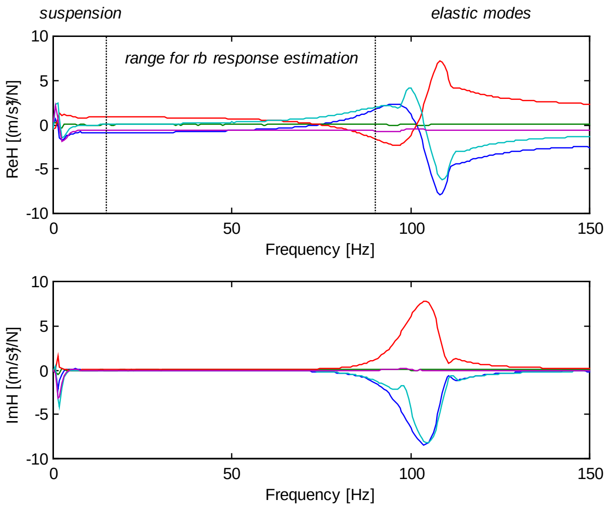
Figure 6: Fan/motor unit

The system has been suspended in soft springs such that the rigid body modes had frequencies lower than 3 Hz. Because of very light damping the influence on the following frequency range has been kept very low so that the real part response has been dominated by the elastic modes of the free/free system from about 10-15 Hz on. This is necessary in order to avoid additional bias when estimating the rigid body response because equation (14) is only meaningful for an ideal free/free system up to the first elastic natural frequency.

Accelerations have been measured at eight locations using sufficiently light accelerometers (3 g) in order not to change the system significantly. The locations have been chosen such that the resulting rigid body mode matrix $\mathbf{X}_R$ had a rank of six which is a necessary condition in order to solve equation (4).

The excitation has been applied using an impulse hammer at three eccentric locations to observe all desired parameters. In order to apply the forces two adapters of around 4 g had to be attached to the structure. A plot of five selected estimated free/free FRFs can be found in figure 7.

Figure 7: Selected free/free FRFs



The estimation has been performed in two steps:

1. estimation of the rigid body response (equation (14)),
2. estimation of rigid body properties (equation (10)),

while the estimation of rigid body properties has been performed without and with known mass (see estimation vectors (6) and (7)) iteratively.

In addition the system with accelerometers and adapters has been tested by pendulum testing and the overall mass has been determined by weighing. Care has been taken that these results are very accurate and thus may serve as reference for comparison.

The results of both tests are listed in the following table. The moments of inertia are listed with respect to the center of gravity.

Table 1: Identification results

Parameter	Pendulum Test and Weighing	Estimation w/o mass [1)]	Deviation [%]
m [kg]	1.689	1.713	1.4
ξ^s [mm]	- [2)]	-0.1	-
η^s [mm]	- [2)]	0.2	-
ζ^s [mm]	57.5	57.8	0.5
$\Theta^s_{\xi\xi}$ [gm²]	2.33	2.36	1.3
$\Theta^s_{\eta\eta}$ [gm²]	2.29	2.20	-3.9
$\Theta^s_{\zeta\zeta}$ [gm²]	2.20	2.16	-1.8
$\Theta^s_{\xi\eta}$ [gm²]	- [2)]	-0.05	-
$\Theta^s_{\xi\zeta}$ [gm²]	- [2)]	-0.02	-
$\Theta^s_{\eta\zeta}$ [gm²]	- [2)]	0.00	-

1) The results (except for the mass) are identical for estimation w/o and with mass.

2) Could not be identified by pendulum testing.

It can be seen that the results correlate very well and that the errors are within the limits given by the accuracy of the test method (i.e. amplitude errors of the pickups, unknown cable masses that contribute to the response, etc.).

4 CONCLUSION

A two step identification method was presented that is capable to provide the complete set of rigid body parameters from free/free FRF test data. The high accuracy achieved is very promising and therefore underlines the value for real applications. However, special care has to be taken for setting up the test: the suspension chosen must not interact significantly with the ideal free/free response of the system and the pickups and exciters must be placed adequately on the system in order to guarantee that all parameters can be observed. If these conditions are met the rigid body properties of an elastic system may be identified.

5 REFERENCES

[BRETL] **Bretl, J./Conti, P.**
Rigid Body Mass Properties From Test Data
Proc. of the 5th Int. Modal Analysis Conference, IMAC; London, England; 1987; P. 655-659

[EWINS] **Ewins, D.J.**
Modal Testing: Theory And Practice
Research Studies Press Ltd.; Taunton, Somerset, England; 1995

[FREGOLENT] **Fregolent, A. et al.**
Identification Of Rigid Body Inertia Properties From Experimental Frequency Response
Proc. of the 10th Int. Modal Analysis Conference, IMAC; San Diego, CA; 1992; P. 279-225

[HAHN] **Hahn, H. et al.**
Inertia Parameter Identification Of Rigid Bodies Using A Multi-Axis Test Facility
Proc. of the 3rd IEEE Conf. on Control Applications; Glasgow; 1994

[HOLZWEISSIG] **Holzweissig, F./Dresig, H.**
Lehrbuch der Maschinendynamik; Grundlagen und praxisorientierte Beispiele; mit 40 Aufgaben mit Lösungen und 53 Tabellen
4., neubearb. Aufl.; Fachbuchverlag; Leipzig; Köln; 1994

[LINK-1] **Link, M.**
Theory Of A Method For Identifying Incomplete System Matrices From Vibration Test Data
Z. Flugwiss. Weltraumforsch. 9, (1985), Heft 2; 1985; P. 76-82

[LINK-2] **Link, M./Qian, G.L.**
Identification Of Dynamic Models Using Base Excitation And Measured Reaction Forces
Revue Française de Mécanique, no. 1994-1

[MANGUS-1] **Mangus, J. et al.**
Direct Estimation Of Rigid Body Properties From Frequency Response Functions
Proc. of the 10th Int. Modal Analysis Conference, IMAC; San Diego, CA; 1992; P. 259-264

[MANGUS-2] **Mangus, J. et al.**
Estimating Rigid Body Properties From Force Reaction Measurements
Proc. of the 11th Int. Modal Analysis Conference, IMAC; Kissimmee, Florida; 1993; P. 469-472

[NAKAMURA] **Nakamura, M. et al.**
Identification Of Inertial Properties For Resilently Supported Mechanical System
DE-Vol. 84-3, 1995 Design Engineering Tech. Conf., Vol. 3 - Part C; Boston, MA; 1995; P. 351-357

[NATKE] **Natke, H. G.**
Einführung in die Theorie und Praxis der Zeitreihen- und Modalanalyse
3., überarb. Aufl.; Vieweg Verlag; Braunschweig; Wiesbaden; 1992

[NUUTILA] **Nuutila, O. et al.**
Experimental Methods For Determining Rigid Body Inertia Properties From Frequency Response Function Of Medium Speed Diesel Engine
Proc. of the 11th Int. Modal Analysis Conference, IMAC; Kissimmee, Florida; 1993; P. 1121-1125

[OKUZUMI] **Okuzumi, H.**
Identification Of The Rigid Body Characteristics Of A Powerplant By Using Experimentally Obtained Transfer Functions
JSAE Review 1991, Society of Automotive Engineering of Japan; 10.1991; P. 28-32

[PANDIT] **Pandit, S. M. et.al.**
Experimental Technique For Accurate Determination Of Rigid Body Characteristics
Proc. of the 10th Int. Modal Analysis Conference, IMAC; San Diego, CA; 1992; P. 307-311

[SCHEDLINSKI-1] **Schedlinski, C./ Link, M.**
Identification Of Frequency Response Functions And Modal Data From Base Excitation Tests Using Measured Interface Forces
Proc. of the 15th ASME Conference on Noise & Vibration; Boston, MA; 1995

[SCHEDLINSKI-2] **Schedlinski, C./ Link, M.**
Identification of Rigid Body Properties using Base Excitation and Measured Interface Forces
Proc. 1996 ESA Conf. on Spacecraft Structures, Materials and Mechanical Testing, European Space Agency, Noordwijk, The Netherlands, 1996

[TOIVOLA] **Toivola, J./Nuutila, O.**
Comparison Of Three Methods For Determining Rigid Body Inertia Properties From Frequency Response Functions
Proc. of the 11th Int. Modal Analysis Conference, IMAC; Kissimmee, Florida; 1993; P. 1126-1132

[URGUEIRA] **Urgueira, A. P. V.**
On The Rigid Body Properties Estimation From Modal Testing
Proc. of the 13th Int. Modal Analysis Conference, IMAC; Nashville, Tennesse; 1995; P. 1479-1483

[WEI] **Wei, Y. S./Reis, J.**
Experimental Determination Of Rigid Body Inertia Properties
Proc. of the 7th Int. Modal Analysis Conference, IMAC; Las Vegas, Nevada; 1989; P. 603-606