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AN APPROACH TO OPTIMAL PICK-UP AND EXCITER PLACEMENT

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ABSTRACT. *This paper introduces an automated approach to pick-up and exciter placement for modal testing purposes and its application to a car body component. It is a two step procedure which in the first step localizes a subset of structural degrees of freedom of an analytical model as measurement points such that the linear independence of the mode shapes to be measured is maximized. In the second step a given number of exciter locations are chosen among the selected measurement points which allow an excitation of the mode shapes. The approach is based on the QR-decomposition of the modal matrix and the QR-decomposition of the product of the mass matrix with the modal matrix.*

NOMENCLATURE

α	force factor
$H(\omega)$	Frequency Response Function
$\Lambda_1(\omega)$	multivariate mode indicator function
ω	circular frequency
$\mathbf{A}$, $\mathbf{E}$, $\mathbf{Q}$, $\mathbf{R}$	matrices
$\mathbf{a}_i$, $\mathbf{r}_i$, $\mathbf{q}_i$	column vectors
Δ	matrix of modal damping values
$\mathbf{F}$	force matrix
$\mathbf{I}$	unity matrix
$\mathbf{M}$	mass matrix
$\mathbf{X}$	modal matrix
$\mathbf{x}_i$	mode shape vector
ω_0^2	matrix of eigenvalues

1 INTRODUCTION

An important goal in modal testing is to perform a test such that the measured mode shapes allow an individual pairing to analysis results. Furthermore all the mode shapes of a test structure in a given frequency band should be excited sufficiently to allow their identification by experimental modal analysis procedures. In order to meet these requirements pick-ups and exciters have to be placed in an appropriate way on the test structure. Although several systematic approaches for automatic selection of pickup and exciter locations have been published earlier the standard approach in practice is still based on the know-how and the skill of the testing personal.

In this paper an automated approach and its application to a prototype structure of a car body component is presented which is aimed at solving the following two problems:

1. Determination of an optimal set of structural degrees of freedom (dof) as measurement points in order to maximize the linear independence of the measured mode shapes (i.e. to increase the spatial resolution of each mode shape with respect to the other mode shapes).

2. Determination of force excitation patterns and their relative magnitudes related to a given number of exciter locations at a sub set of the chosen measurement points that allow an excitation of all the mode shapes contained in a given frequency band.

The approach presented here uses a QR-decomposition technique to determine the most effective subsets of the modal matrix and the force excitation matrix of an appropriate analytical model. An application is presented for the case of a car body component where the results are in contrast to empirical expectations.

2 THEORY

2.1 Principles

The basic assumption shall be that a given analytical model already describes the real structure in an appropriate manner, i.e. the calculated mode shapes do not differ too significantly from the ones to be measured.

In order to determine an optimal subset of structural degrees of freedom that can be used as measurement points (i.e. pick-up locations) the modal matrix is investigated. The idea is that 'the most linear independent' rows of the modal matrix indicate degrees of freedom that should be chosen as pick-up locations because they form the smallest possible modal matrix which provides a MAC (Modal Assurance Criteria) matrix with minimized off diagonal terms (i.e. the ability to distinguish between similar mode shapes is enhanced). The technique to extract those rows is presented in the following chapters.

The same logic applies to the selection of degrees of freedom used as excitation points. The only difference here is that the product of the mass matrix with the modal matrix taken at the chosen pick-up locations is investigated instead. This heuristic approach is based on the fact that an excitation pattern proportional to the product of the mass matrix with a given mode always excites this very mode due to the orthogonality of the modal matrix with respect to the mass matrix. If we now extract those rows again that are 'the most linear independent' we very likely obtain exciter locations that allow a (sub-optimal) excitation of all mode shapes assembled in the modal matrix.

2.2 The QR-Decomposition

A suitable method to extract the *columns* of a matrix that are 'the most linear independent' is the QR-decomposition [3] also used in [1] to determine optimal pick-up locations.

Suppose the matrix $\mathbf{A}$ is given. Then the QR-decomposition is given by

$$\mathbf{A\,E} = \mathbf{Q\,R} \tag{1}$$

with

- $\mathbf{A} \in \mathrm{R}^{m,n}$, a given matrix
- $\mathbf{Q} \in \mathrm{R}^{m,m}$, an orthogonal matrix ($\mathbf{Q}^T\mathbf{Q} = \mathbf{I}$)
- $\mathbf{R} \in \mathrm{R}^{m,n}$, an upper triangular matrix with decreasing diagonal elements
- $\mathbf{E} \in \mathrm{R}^{n,n}$, a permutation matrix that exchanges columns of $\mathbf{A}$

Due to the characteristics of the QR-decomposition the first columns of ($\mathbf{A\,E}$) are those that are the 'most linear independent' columns. In order to show this let $\tilde{\mathbf{A}} = \mathbf{A\,E}$. According to equation (1) we can write:

$$\tilde{\mathbf{A}} = \mathbf{Q\,R}$$

The first column is represented by

$$\tilde{\mathbf{a}}_1 = \mathbf{Q\,r}_1 = \mathbf{q}_1\,\mathrm{R}_{11}$$

$$\tilde{\mathbf{a}}_j = \mathbf{Q\,r}_j = \mathbf{q}_1\,\mathrm{R}_{1j} + \mathbf{q}_2\,\mathrm{R}_{2j} + \ldots + \mathbf{q}_j\,\mathrm{R}_{jj} \tag{2}$$

$$\tilde{\mathbf{a}}_{j+1} = \mathbf{Q\,r}_{j+1} = \mathbf{q}_1\,\mathrm{R}_{1(j+1)} + \mathbf{q}_2\,\mathrm{R}_{2(j+1)} + \ldots + \mathbf{q}_{j+1}\,\mathrm{R}_{(j+1)(j+1)} \tag{3}$$

where $\mathbf{r}_j$ and $\mathbf{q}_j$ denote the j-th columns of $\mathbf{R}$ respectively $\mathbf{Q}$ and R_{ij} denotes the element in the i-th row and the j-th column of $\mathbf{R}$.
Due to the fact that the columns of $\mathbf{Q}$ form an orthogonal basis of the space of the columns of $\tilde{\mathbf{A}}$, equations (2) and (3) show that columns 1 to (j+1) are linear dependent only if $\mathrm{R}_{(j+1)(j+1)} = 0$. The magnitude of the value of $\mathrm{R}_{(j+1)(j+1)}$ can thus be considered as an indicator for the linear independence of the first (j+1) columns of the matrix $\tilde{\mathbf{A}}$.

Thus, considering the fact that the diagonal elements of $\mathbf{R}$ are arranged in the descending order of their absolute values, the first columns of $\tilde{\mathbf{A}}$ are those that show the strongest linear independent characteristics.

It is evident that the described method always yields a maximum of n diagonal values R_{ii}. The selection of $s > n$ columns however can be performed in the following way:

1. Perform a QR-decomposition on $\mathbf{A}$ and select the first n columns of $\mathbf{A\,E}$ (the column numbers with respect to $\mathbf{A}$ can be extracted from $\mathbf{E}$). After this set the selected columns to zero.

2. Perform a QR-decomposition on the modified matrix $\mathbf{A}^2$ and select min [n, s-n] columns. After this again set the selected columns to zero.

3. Repeat 2. until s columns are selected.

2.3 Optimal Pick-Up and Exciter Placement

As already assumed we have access to a mathematical model which provides the following two matrices:

- $\mathbf{X} = [\mathbf{x}_1,\mathbf{x}_2,\ldots,\mathbf{x}_m] \in \mathrm{R}^{n,m}$, a modal matrix assembled from m mode shapes with n components (i.e. dof) each ($n \geq m$)
- $\mathbf{M} \in \mathrm{R}^{n,n}$, a mass matrix

In order to select s optimal pick-up locations the QR-decomposition is applied to the *transpose* of the modal matrix $\mathbf{X}$. The transpose is taken because the information for one given degree of freedom is provided in one single row of the modal matrix and the QR-decomposition does only sort the columns and not the rows of a matrix.

Now the question remains how to choose s. Since we have a (n,m) modal matrix and $n \geq m$ the rank of the modal matrix is m, too. Thus we have to restrict ourselves to the selection of s = m pick-up locations (If $s > m$ is chosen the additional $s - m$ pick-up locations are to be found in the vicinity of the first m ones which will not improve the measurement information.). After this the MAC matrices of the modal matrix **X** versus itself and the modal matrix reduced to the selected pick-up degrees of freedom $\tilde{\mathbf{X}}$ versus itself can be calculated according to (4).

$$\left[\mathrm{MAC}_{ij}\right] = \frac{\left(\mathbf{x}_i^T \mathbf{x}_j\right)^2}{\left\|\mathbf{x}_i\right\| \left\|\mathbf{x}_j\right\|} \quad ; \; i, j = 1 \ldots m \qquad (4)$$

The comparison of the off diagonal terms of both MAC matrices can be used to validate the selection result.

The optimal exciter locations can be obtained simply by applying the QR-decomposition to the *transpose* of the exciter force matrix $\mathbf{F} = (\mathbf{M}\,\mathbf{X}) \in R^{n,m}$ reduced to the chosen pick-up degrees of freedom $\mathbf{F}_{red} \in R^{m,m}$ (This reduction has to be made in order to arrive at exciter locations that coincide with the chosen pick-up degrees of freedom - although the result may not be optimal.). Here, the number s of degrees of freedom to be selected is determined by the number of exciters n_e that are to be used for the test which is limited by the available test equipment.

Theoretically, in case of proportional damping, the exciter force vector $\mathbf{F}_i = (\mathbf{M}\,\mathbf{X}_i)$ (i.e. the i-th column of **F**) excites only the i-th mode shape $\mathbf{X}_i$ while the remaining mode shapes $\mathbf{X}_j$ ($j \neq i$) are not excited. This can be seen from the equation of motion transformed to modal degrees of freedom q_i ($i = 1, 2, ..., n$) which is decoupled into n independent equations (5).

$$(-\,\omega^2\,\mathbf{I} + \omega_o^{\,2} + j\,\omega\,\Delta)\,\mathbf{q} = \mathbf{X}^T\,\mathbf{F} = \mathbf{X}^T\,\mathbf{M}\,\mathbf{X} = \mathbf{I} \qquad (5)$$

with

- $\omega_o^{\,2} = \mathrm{diag}\,(\omega_{o1}^{\,2}, \omega_{o2}^{\,2}, ..., \omega_{on}^{\,2})$ matrix of eigenvalues
- $\Delta = \begin{bmatrix} \Delta_{11} & \Delta_{12} & \cdots \\ \Delta_{21} & \Delta_{22} & \\ \vdots & & \ddots \end{bmatrix}$

  = diag in case of proportional damping
  ≠ diag in case of non-proportional damping

The assumption of proportional damping represents a good approximation the more the better the damping values decrease and the distance of neighbored eigenfrequencies increases.

After the selection of m pick-up and n_e exciter locations the validity of the approach may be checked by calculating the multivariate mode indicator function from synthesized frequency response functions [2, 4] solving the eigenproblem (6) for each eigenfrequency of interest.

$$[-(\mathbf{H}^T_{re}\,\mathbf{H}_{re} + \mathbf{H}^T_{im}\,\mathbf{H}_{im})\,\Lambda_1(\omega_{0i}) + \mathbf{H}^T_{re}\,\mathbf{H}_{re}]\,\alpha(\omega_{0i}) = \mathbf{0} \qquad (6)$$

with

ω_{0i}	eigenfrequency (i = 1...m)
$\Lambda_1(\omega_{0i})$	smallest eigenvalue = multivariate mode indicator function value at $\omega = \omega_{0i}$
$\mathbf{H} \in R^{n,ne}$	FRF matrix (re = real-, im = imaginary part)
$\alpha(\omega_{0i}) \in R^{ne,1}$	eigenvector = appropriate force vector

This multivariate mode indicator function $\Lambda_1(\omega)$ shows explicitly whether all desired mode shapes can be excited or not. Furthermore the calculation of the multivariate mode indicator function provides i = 1, 2, ..., m excitation force vectors. Each of them allows the best possible excitation for the associated mode shape with n_e exciter locations.

3 APPLICATION: CAR BODY COMPONENT

3.1 Aluminum Car Body Component

The described methods for selecting optimal pick-up and exciter locations were applied to a model of an aluminum car body component (figure 1).

The Finite Element Model of the car body component consists of:

- 168 elements and
- 132 nodes.

The calculation of the mode shapes has been performed using a symmetric model under free/free condition. The Finite Element Analysis yielded 19 mode shapes (including three rigid body modes) in a frequency range from zero up to 210 Hz.

3.2 Optimal Pick-Up Placement

As already discussed the modal matrix was to be investigated here. The rows of the modal matrix related to rotational degrees of freedom had to be neglected because rotational degrees of freedom cannot be measured. Thus the modal matrix was reduced to a (396,19) matrix. The QR-decomposition led to 19 pick-up degrees of freedom presented in figure 2. The numbers in figure 2 indicate the ranking of the degrees of freedom coming from the QR-decomposition.

Figure 1: Survey of car body component.

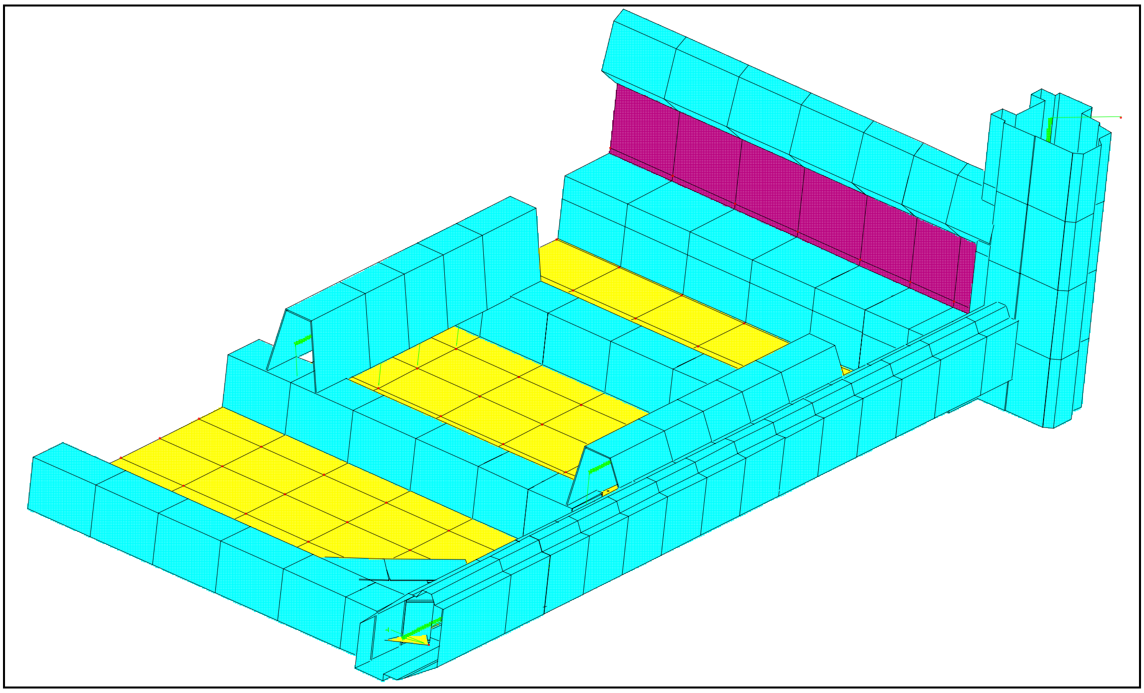
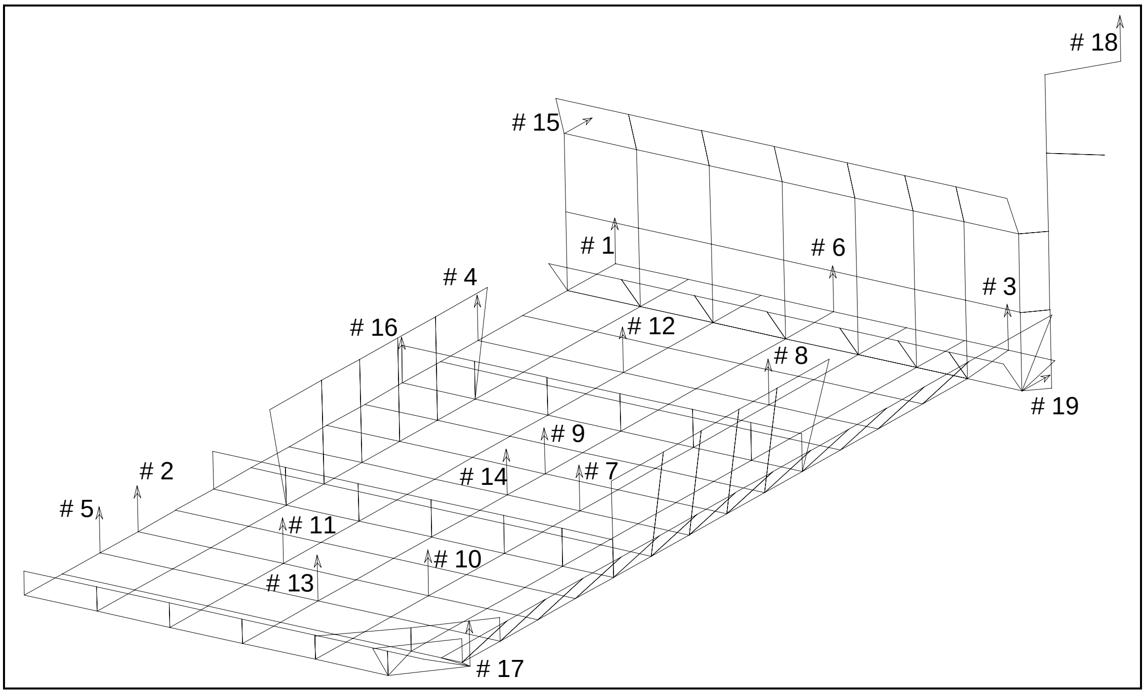


Figure 2: 19 selected pick-up locations.



The MAC matrices show a similar behavior for the full (figure 3) and the reduced modal matrix (figure 4). This is a good indicator for an appropriate choice of pick-up degrees of freedom.

However, the MAC matrix with respect to the reduced modal matrix exhibits some larger off-diagonal terms than the one related to the full modal matrix and this means that the linear independence of the reduced modes is worse than the one of the modes using all degrees of freedom. Yet, since the off-diagonal terms are still very small and not larger than about 0.3 this does not introduce any problem.

Figure 3: MAC matrix - all degrees of freedom.

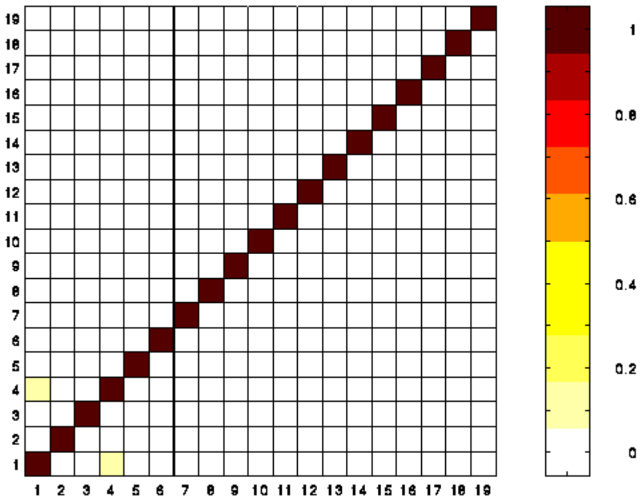
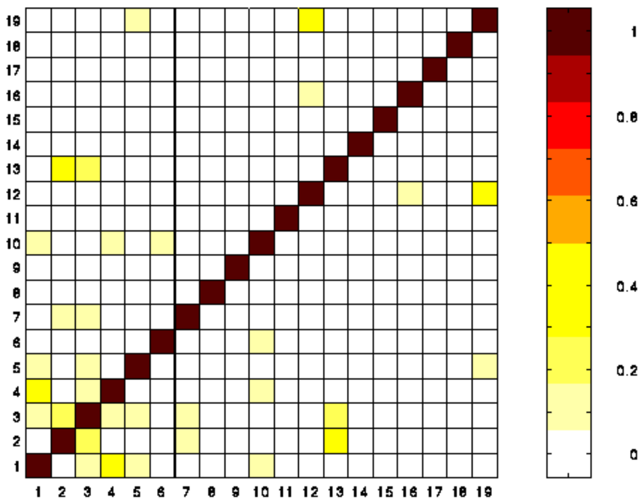


Figure 4: MAC matrix - 19 selected degrees of freedom.



3.3 Optimal Exciter Placement

Here the force matrix $\mathbf{F} = (\mathbf{M}\ \mathbf{X})$ was investigated. Since only the 19 degrees of freedom chosen above were considered this matrix was reduced to size (19,19). Furthermore it was defined that the number of exciters used for the test should be as small as possible.

In order to determine the appropriate number of exciter locations the number of exciters was increased step by step while the exciter degrees of freedom where chosen with respect to their ranking coming from the QR-decomposition of $(\mathbf{F}_{red})^T$. For each exciter configuration the resulting multivariate mode indicator function was inspected then.

It showed that for two exciters (#1 and # 2 in figure 7) 18 of the 19 modes could already be excited significantly - only mode 17 could not be excited at all (figure 5).

The multivariate mode indicator functions did not change significantly after adding one or two additional exciters. However adding a fifth exciter (# 3 in figure 7) finally allowed a perfect excitation of the missing mode and it was therefore concluded, that this exciter location was crucial for the excitation of this very mode.

Because all the other modes could already be excited using two exciters and mode 17 could be excited only after adding the fifth exciter location, the third and the fourth exciter location have been removed from the set again. The remaining three exciter locations (see figure 7) now provided the desired minimum set of exciter locations capable of exciting all the 19 modes in the given frequency band (see figure 6, a zero in the multivariate mode indicator function indicates a perfect excitation of the corresponding mode).

A subsequent visual inspection of mode 17 displayed that it was a local mode of one single plate section which could not be excited using the first four exciter locations coming from the QR-decomposition of $(\mathbf{F}_{red})^T$ and the exciter added to primarily excite mode 17 was found to be situated in the vicinity of the maximum amplitude of mode 17 !

Figure 5: Multivariate mode indicator function - two exciters.

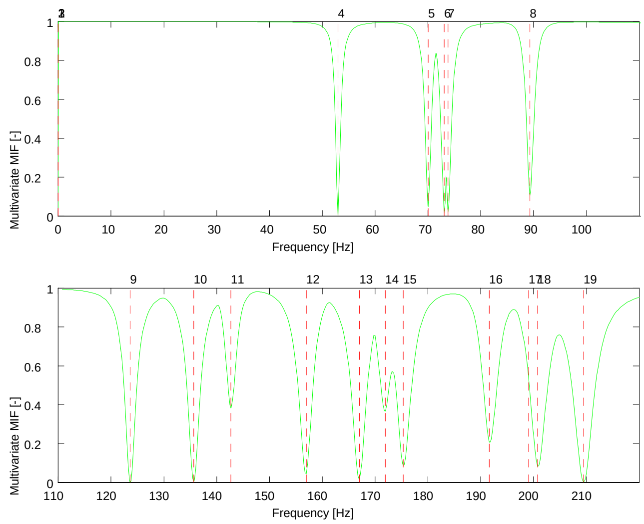
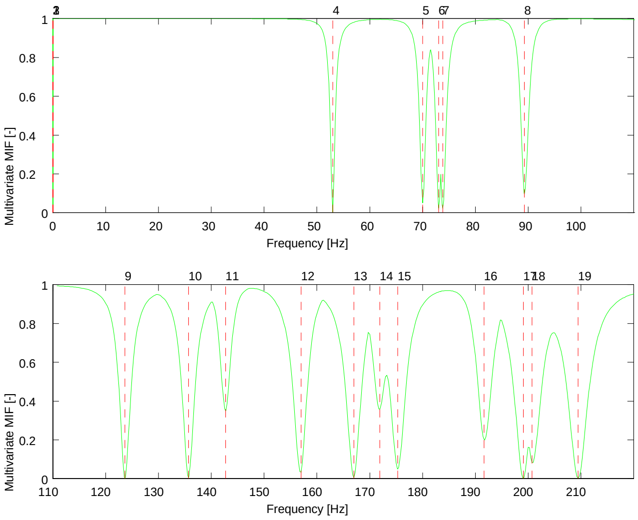
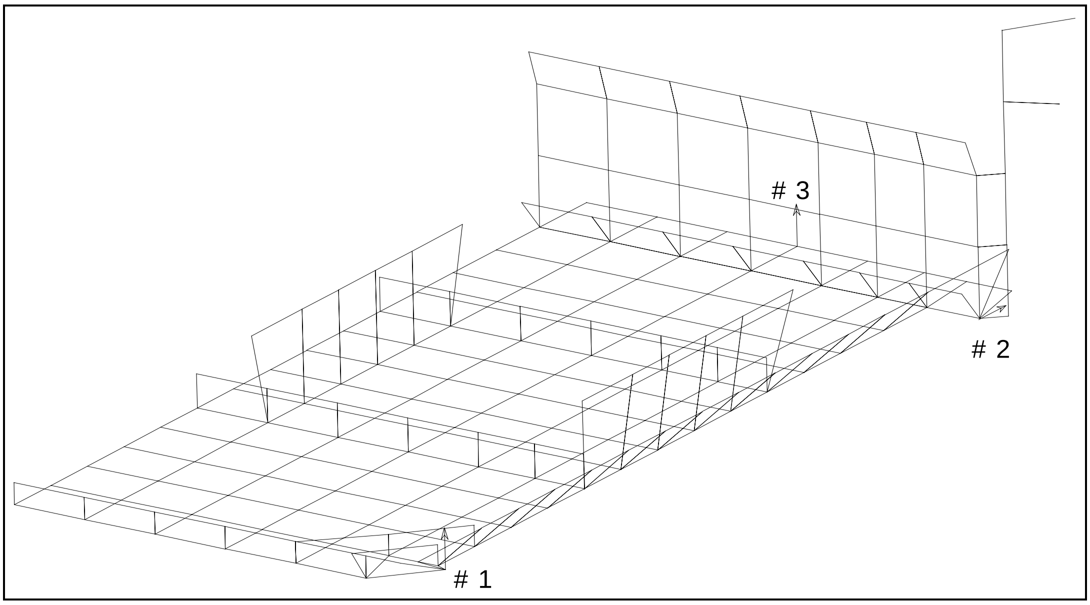


Figure 6: Multivariate mode indicator function - three exciters.



It can be seen from figure 6 that all modes can be excited sufficiently using the three selected exciter positions if the appropriate multi-point excitation force pattern α is provided. The three selected exciter positions therefore are very well chosen and should be considered in a real test. Furthermore it should be noted that the resulting exciter positions are quite surprising and would not have been expected by simple inspection of the 19 modes.

Figure 7: Three selected exciter degrees of freedom.



It should be noted that a single-point excitation applied at either one of the three selected exciter position does not yield the same good results. In the case that only single-point excitation can be used due to restrictions of the test equipment the following procedure is proposed:

1. Collect frequency response functions (FRFs) from exciting the structure with one modal exciter at all three selected exciter positions subsequently or use impulse hammer excitation with $n_e = 3$ reference pick-ups at these exciter positions.

2. Superimpose the three collected FRFs using the force factors $\alpha_j^{(k)}$ from the multivariate mode indicator function calculation according to (7).

$$\tilde{H}_i^{(k)}(\omega) = \sum_j \alpha_j^{(k)} H_{ij}(\omega) \tag{7}$$

i = 1, 2, ..., m = 19 → number of response dofs
j = 1, 2, ..., n_e = 3 → number of exciter dofs
k = 1, 2, ..., m = 19 → number of mode shapes

$\widetilde{H}_i^{(k)}(\omega)$ → multi-point excitation FRF for the k-th mode shape

$H_{ij}(\omega)$ → measured FRF at response dof i and single-point excitation dof j

3. Extract the k mode shapes by standard curve fitting procedures from the corresponding k FRFs $\widetilde{H}_i^{(k)}(w)$.

(Note: The procedure depicted above is only valid for linear structures and should not be applied to structures which exhibit significant non-linear behavior.)

4 CONCLUSIONS

The preceding chapters showed one possible approach for pick-up and exciter placement that may be used to optimize the test set-up. Because it takes analytic data into account it pays respect to the dynamic behavior of a given structure which is an advantage to pure intuition or know-how of testing personal. Nevertheless there are inherent problems that should not be forgotten:

- The results can only be as good as the initial analytic model allows.
- The selected pick-up and/or exciter positions may not be accessible and thus compromises may have to be considered.
- Due to the QR-decomposition the selected pick-up locations may not provide enough information to visualize the mode shapes in a sufficient way (therefore more, e.g. evenly distributed pick-ups, should be used for the test which may be neglected afterwards since these additional pick-ups may increase the linear dependency of the mode shapes again).

In the presented case however the results are very encouraging: with only 19 pick-up and three exciter locations every mode in the given frequency band may be excited accurately and the linear independence of the modes is excellent. Even if the third exciter was missing still 18 of the 19 mode shapes could be excited.

The presented approach therefore seems to provide reliable results and should be considered as a basis for setting-up a test.

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