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IDENTIFICATION OF RIGID BODY PROPERTIES USING BASE EXCITATION AND MEASURED INTERFACE FORCES

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ABSTRACT

Base excitation testing on a shaking table is frequently used in industry to qualify mechanical systems with respect to specified base acceleration levels. The measurement of structural and interface (base) accelerations furthermore allows an identification of the natural frequencies, mode shapes and modal damping values of the fixed system. However, modal masses, mass participation factors and effective masses cannot be identified.

Now, the additional measurement of the interface forces allows not only to identify these missing data but also the modal data of the free system and the rigid body properties.

This paper describes an approach to identify the rigid body properties (overall mass, moments of inertia and location of center of gravity) from base excitation test data since the other possibilities have already been published by the authors elsewhere. The requirements, capabilities and restrictions of the approach will be discussed in detail and an analytical example will be presented.

Keywords: rigid body properties, identification, base excitation, shaking table, interface forces

1 INTRODUCTION

Base excitation testing on a shaking table is frequently used in industry to qualify mechanical systems with respect to specified base acceleration levels. These tests are usually run separately for the axial and the lateral directions. Structural and interface (base) accelerations are measured and the modal data of the fixed system except for modal masses, mass participation factors and effective masses can be identified from the test data.

If the interface forces are measured in addition to the interface accelerations some new possibilities for data extraction arise. In this case the following data can be identified additionally:

- The modal masses, mass participation factors and effective masses of the fixed system.
- The modal data of the free system (from frequency response functions of the free system that can be estimated with respect to unit interface forces).
- The rigid body properties (overall mass, moments of inertia and location of center of gravity).

Here only the third possibility, the extraction of the rigid body properties from base excitation test data, shall be presented. The other two possibilities have already been published by the authors earlier ([LINK-3, SCHEDLINSKI]).

Until now the identification of rigid body properties from vibration test data has almost completely been restricted to cases where the data are assembled from a test of the system under free boundary conditions. The approach presented here is a frequency domain method based on the equation of motion of an unrestrained rigid body that can be applied if acceleration and interface force data from a base excitation test are measured. For a rigid body the structural accelerations as well as the interface forces may be transformed with respect to an arbitrarily chosen reference point. The rigid body properties can then be identified with respect to this very point.

It is an advantage of this approach that it can be applied to elastic structures also because it is possible to separate the rigid body response from the elastic response in the frequency domain. A disadvantage may be that for identification of the complete 3-dimensional set of rigid body properties six linear independent interface acceleration vectors must be generated which is only possible on six-axis shaking tables. However, if only the three linear independent interface acceleration vectors are used that can be produced using an uniaxial shaking table the overall mass and the location of the center of gravity with respect to the chosen reference point may still be identified.

2 PRESENT METHODS FOR RIGID BODY PROPERTY IDENTIFICATION

The present methods for rigid body property identification may be divided into two main categories:

- time domain methods
- frequency domain methods

The first *time domain methods* to mention are the classical static methods and pendulum methods which are still commonly used [HOLZWEISSIG]. They provide reliable results within a very short testing time, if performed accurately. Of course it would be desirable to avoid these additional tests by extracting the rigid body properties from vibration tests (e.g. shaking table tests) if they are executed anyway.

Other time domain methods are based on the evaluation of vibration test data. Pandit, et. al [PANDIT] for instance focus on the time domain equation of motion of a rigid body under elastic, damped mounting conditions. Hahn et. al [HAHN] use the time domain test data of a six-axes shaking table system where the exciter forces are measured in addition to the acceleration responses.

Advantage of the time domain methods is the direct evaluation of the test data without the necessity of a transformation into frequency domain and the signal processing problems coming with it. The identification algorithm can directly be applied to the test data. A disadvantage is, however, that if the system under observation does not behave as a rigid body in the excited frequency range low pass filtering of the test data must be performed. This requires an additional analysis of the frequency content of the test data. Furthermore the influence of the structure's elasticity on the response in the used frequency range may not be eliminated by low pass filtering alone if the first elastic natural frequency of the system is very low. In this case time domain methods cannot be applied at all unless special account is taken for such effects.

Frequency domain methods on the other hand bear the possibility to circumvent this disadvantage of the time domain methods because a separation of rigid and elastic system behavior is possible even if the first elastic natural frequency is very low. The frequency domain methods may be subdivided into three categories:

- modal parameter methods
- methods of direct physical parameter identification
- residual inertia methods (massline methods)

The modal parameter methods are based on the orthogonality relation between the mass matrix of the system and the rigid body modes (see e.g. [BRETL]). Advantage of these methods is that they use the results of a preceding experimental modal analysis of the quasi free system (suspended in soft springs). The experimental modal analysis itself may be performed using various well established methods (see e.g. [EWINS, NATKE]). A disadvantage is that in general not all rigid body modes may be excited in a real test.

The methods of direct physical parameter identification focus on a fit of system matrices to identified frequency response functions. General methods take into account the elastic behavior of the system [LINK-1, LINK-2] which does not restrict them to ideal rigid systems. A disadvantage is that, as with the modal parameter methods above, in general not all rigid body modes of the quasi free system may be excited in a real test.

Special methods of direct physical parameter identification are based on a fit of the equation of motion of a rigid body under elastic, damped mounting conditions to identified frequency response functions ([MANGUS-1, NAKAMURA]). A new approach [MANGUS-2] proposes the measurement of the vibration response of a fixed system under base excitation with an additional measurement of the interface forces. Advantage of the special methods is their simple formulation which is better suited for the given identification problem. A disadvantage is again the requirement of a rigid body system behavior. However in the frequency domain a separation of the rigid and the elastic behavior is possible and merely increases the analysis effort.

The residual inertia methods (massline methods) which have been under investigation in many publications recently may be regarded as a special case of the special methods of direct physical parameter identification. Basis for these methods is the equation of motion of a rigid body under free boundary conditions with respect to a given reference point. Input for these methods are residual inertiae which can be extracted from vibration test data in various ways.

Bretl and Conti [BRETL] have been one of the first to publish a residual inertia method. They extract the residuals directly from frequency response functions of the system in a low frequency suspension using the frequency range between the highest rigid body mode and the first elastic mode (massline). Wei and Reis [WEI] identify the residual inertiae with a special curve fitting procedure. Here the residual inertiae are a by-product of the modal identification of the first elastic mode. Okuzumi [OKUZUMI] proposes an iterative method in order to identify the rigid body properties with respect to the center of gravity which has the advantage, that the identified rigid body properties do not have to be transformed to the center of gravity in a subsequent step. A thorough investigation of the identification equation itself can be found in [FREGOLENT].

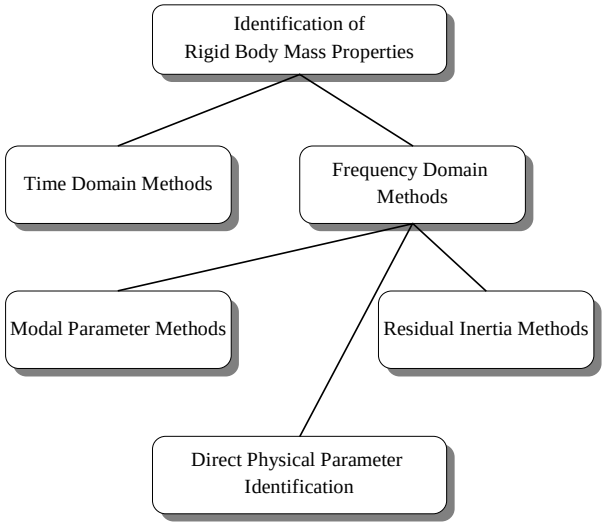

Figure 1: Overview of identification methods

In general the frequency domain methods seem to have the highest development level in the literature. Toivola and Nuutila [TOIVOLA, NUUTILA] have performed an

analytical and an experimental study where they compared a modal parameter method, a method of direct physical parameter identification and a residual inertia method. The residual inertia method provided the most accurate results the modal parameter method the second best and the method of direct physical parameter identification the poorest. Figure 1 above gives an overview of the discussed methods.

3 EQUATIONS OF MOTION & TRANSFORMATION OF ACCELERATIONS AND FORCES

The linearized equations of motion of an unrestrained rigid body (see figure 2) written down for an arbitrary reference point A yields equation (1). It is entirely equivalent to the equation of motion of a rigid body mounted for example at reference point A on a rigid (six-axes) shaking table. The accelerations and forces/moments are in this case the interface (base) accelerations and forces/moments at point A.

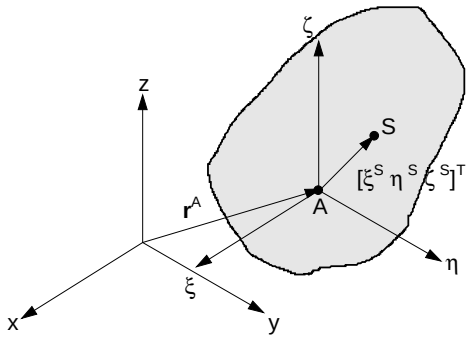

Figure 2: Unrestrained rigid body

$$\underbrace{\begin{bmatrix} m & 0 & 0 & 0 & m\zeta^S & -m\eta^S \\ 0 & m & 0 & -m\zeta^S & 0 & m\xi^S \\ 0 & 0 & m & m\eta^S & -m\xi^S & 0 \\ 0 & -m\zeta^S & m\eta^S & \Theta^A_{\xi\xi} & -\Theta^A_{\xi\eta} & -\Theta^A_{\xi\zeta} \\ m\zeta^S & 0 & -m\xi^S & -\Theta^A_{\xi\eta} & \Theta^A_{\eta\eta} & -\Theta^A_{\eta\zeta} \\ -m\eta^S & m\xi^S & 0 & -\Theta^A_{\xi\zeta} & -\Theta^A_{\eta\zeta} & \Theta^A_{\zeta\zeta} \end{bmatrix}}_{\mathbf{M}^A} \underbrace{\begin{bmatrix} \ddot{\xi}^A \\ \ddot{\eta}^A \\ \ddot{\zeta}^A \\ \ddot{\alpha}^A \\ \ddot{\beta}^A \\ \ddot{\gamma}^A \end{bmatrix}}_{\mathbf{a}^A} = \underbrace{\begin{bmatrix} f^A_\xi \\ f^A_\eta \\ f^A_\zeta \\ t^A_\xi \\ t^A_\eta \\ t^A_\zeta \end{bmatrix}}_{\mathbf{f}^A} \tag{1}$$

with:

$\mathbf{M}^A$	rigid body mass matrix w.r.t. point A
$\mathbf{a}^A$	vector of accelerations of point A
$\mathbf{f}^A$	vector of forces/moments at point A
m	overall mass
ξ^S, η^S, ζ^S	location of center of gravity w.r.t. point A
$\Theta^A_{\xi\xi}$, ...	moments of inertia w.r.t. point A
$\ddot{\xi}^A$, ...	translational accelerations of point A
$\ddot{\alpha}^A$, ...	circular accelerations about point A
f^A_ξ, ...	forces at point A
t^A_ξ, ...	moments at point A

In general not all translational and rotational accelerations or forces and moments respectively can be measured for the chosen reference point under real test conditions. Therefore translational accelerations and forces are measured at as many accessible locations as desired and subsequently transformed on the reference point. For a rigid body the needed transformation is purely geometric.

Supposing that the displacements $\mathbf{u}^A$ at the reference point A are known the translational displacements $\mathbf{u}^P$ at a given point P can be derived by a simple linear combination (see figure 3 and equation (2)).

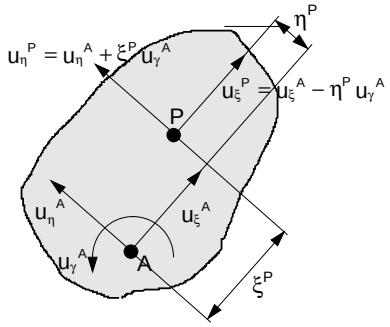

Figure 3: Transformation of displacements (2D)

$$\underbrace{\begin{bmatrix} u_\xi^P \\ u_\eta^P \\ u_\zeta^P \end{bmatrix}}_{\mathbf{u}^P} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & \zeta^P & -\eta^P \\ 0 & 1 & 0 & -\zeta^P & 0 & \xi^P \\ 0 & 0 & 1 & \eta^P & -\xi^P & 0 \end{bmatrix}}_{\mathbf{X}_R} \underbrace{\begin{bmatrix} u_\xi^A \\ u_\eta^A \\ u_\zeta^A \\ u_\alpha^A \\ u_\beta^A \\ u_\gamma^A \end{bmatrix}}_{\mathbf{u}^A} \tag{2}$$

Matrix $\mathbf{X}_R$ contains the components of the rigid body modes for point P with respect to the reference point A. Now let $\mathbf{u}^M$ be a vector of displacements at m degrees of freedom. If the corresponding components of the rigid body modes are used to assemble the rows of matrix $\mathbf{X}_R$ we arrive at (3-a) which is an overdetermined system of equations in case of m > 6:

$$\underbrace{\mathbf{u}}_{(m,1)}^M = \underbrace{\mathbf{X}_R}_{(m,6)} \underbrace{\mathbf{u}}_{(6,1)}^A \tag{3-a}$$

Since $\mathbf{X}_R$ is constant equation (3-a) also holds for the accelerations:

$$\ddot{\mathbf{u}}^M = \mathbf{X}_R\, \ddot{\mathbf{u}}^A \Rightarrow \underbrace{\mathbf{a}}_{(m,1)}^M = \underbrace{\mathbf{X}_R}_{(m,6)} \underbrace{\mathbf{a}}_{(6,1)}^A \tag{3-b}$$

Equation (3-b) can now be solved in a weighted least squares sense. Therefore a constant diagonal (m,m) weighting matrix **W** can be introduced (see equation (4)). If no special ranking of the measured accelerations is intended the unity matrix is taken. However, further below a strategy how to choose **W** is presented that improves the results.

$$\mathbf{a}^A = \left(\mathbf{X}_R^T\,\mathbf{W}\,\mathbf{X}_R\right)^{-1}\mathbf{X}_R^T\,\mathbf{W}\,\mathbf{a}^M \tag{4}$$

The relation between the force resultants $\mathbf{f}^A$ and the forces applied at m measurement degrees of freedom $\mathbf{f}^M$ is obtained from the equilibrium conditions expressed by the principle of virtual work: The virtual work of the force resultants and their corresponding displacements at point A must be equal to the virtual work of the applied forces and the displacements at the corresponding measurement degrees of freedom:

$$\left(\mathbf{f}^A\right)^T\delta\mathbf{u}^A \overset{!}{=} \left(\mathbf{f}^M\right)^T\delta\mathbf{u}^M$$

Using the relation between the displacements at point A and those at the m measured degrees of freedom (3-a) the following equilibrium equation is obtained:

$$\underbrace{\mathbf{f}^A}_{(6,1)} = \underbrace{\mathbf{X}_R^T}_{(6,m)}\,\underbrace{\mathbf{f}^M}_{(m,1)} \tag{5}$$

4 IDENTIFICATION EQUATIONS

Basis for the development of the identification equations are the equations of motion (1). Since only 10 quantities in (1) are unknown the following estimation vector σ can be defined (see e.g. [URGUEIRA]).

$$\sigma = \left[m \;\; m\xi^S \;\; m\eta^S \;\; m\zeta^S \;\; \Theta^A_{\xi\xi} \;\; \Theta^A_{\eta\eta} \;\; \Theta^A_{\zeta\zeta} \;\; \Theta^A_{\xi\eta} \;\; \Theta^A_{\xi\zeta} \;\; \Theta^A_{\eta\zeta}\right]^T \tag{6}$$

A disadvantage of this estimation vector is that the location of the center of gravity cannot be explicitly estimated so that the error on the estimated overall mass and the error on the coupled terms $m\xi^S$, ... may add. However, an explicit estimation is possible if the overall mass has a priori been determined (e.g. by weighing). Then the number of unknowns is reduced to 9 and the estimation vector does not contain the overall mass (see e.g. [BRETL]). Another possibility is to introduce the estimation vector

$$\sigma = \left[\Delta \;\; \xi^S \;\; \eta^S \;\; \zeta^S \;\; \Theta^A_{\xi\xi} \;\; \Theta^A_{\eta\eta} \;\; \Theta^A_{\zeta\zeta} \;\; \Theta^A_{\xi\eta} \;\; \Theta^A_{\xi\zeta} \;\; \Theta^A_{\eta\zeta}\right]^T \tag{7}$$

Reassembling (1) then yields:

$$\underbrace{\begin{bmatrix} m\ddot{\xi}^A & 0 & -m\ddot{\gamma}^A & m\ddot{\beta}^A & 0 & 0 & 0 & 0 & 0 & 0 \\ m\ddot{\eta}^A & m\ddot{\gamma}^A & 0 & -m\ddot{\alpha}^A & 0 & 0 & 0 & 0 & 0 & 0 \\ m\ddot{\zeta}^A & -m\ddot{\beta}^A & m\ddot{\alpha}^A & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & m\ddot{\zeta}^A & -m\ddot{\eta}^A & \ddot{\alpha}^A & 0 & 0 & -\ddot{\beta}^A & -\ddot{\gamma}^A & 0 \\ 0 & -m\ddot{\zeta}^A & 0 & m\ddot{\xi}^A & 0 & \ddot{\beta}^A & 0 & -\ddot{\alpha}^A & 0 & -\ddot{\gamma}^A \\ 0 & m\ddot{\eta}^A & -m\ddot{\xi}^A & 0 & 0 & 0 & \ddot{\gamma}^A & 0 & -\ddot{\alpha}^A & -\ddot{\beta}^A \end{bmatrix}}_{\mathbf{B}} \underbrace{\begin{bmatrix} \Delta \\ \xi^S \\ \eta^S \\ \zeta^S \\ \Theta^A_{\xi\xi} \\ \Theta^A_{\eta\eta} \\ \Theta^A_{\zeta\zeta} \\ \Theta^A_{\xi\eta} \\ \Theta^A_{\xi\zeta} \\ \Theta^A_{\eta\zeta} \end{bmatrix}}_{\sigma} = \underbrace{\begin{bmatrix} f^A_\xi \\ f^A_\eta \\ f^A_\zeta \\ t^A_\xi \\ t^A_\eta \\ t^A_\zeta \end{bmatrix}}_{\mathbf{f}^A} \tag{8}$$

Here the accelerations and the overall mass form the measurement matrix $\mathbf{B}$ and the forces and moments form the force vector $\mathbf{f}^A$. For an ideal estimation Δ must be equal to one. For a non ideal estimation the deviation from one may be used as an indicator for the quality of the estimation. Equation (8) is now assembled for $i = 1...n_e$ excitation configurations such that the related measurement matrices $\mathbf{B}_{(i)}$ and the force vectors $\mathbf{f}^A_{(i)}$ form the following overdetermined equation system:

$$\underbrace{\begin{bmatrix} \mathbf{B}_{(1)} \\ \mathbf{B}_{(2)} \\ \vdots \\ \mathbf{B}_{(n_e)} \end{bmatrix}}_{\hat{\mathbf{B}}} \sigma = \underbrace{\begin{bmatrix} \mathbf{f}^A_{(1)} \\ \mathbf{f}^A_{(2)} \\ \vdots \\ \mathbf{f}^A_{(n_e)} \end{bmatrix}}_{\hat{\mathbf{F}}} \tag{9}$$

Equation (9) can be solved in a least squares sense only if a minimum of two linear independent excitation configurations (interface acceleration vectors) have been measured leading to 12 equations for the 10 unknowns. These excitation configurations must be chosen such that all the parameters to be identified can sufficiently be observed.

$$\sigma = \left(\hat{\mathbf{B}}^T\hat{\mathbf{B}}\right)^{-1}\hat{\mathbf{B}}^T\,\hat{\mathbf{F}} \tag{10}$$

An important reduction of (8) is derived if only translational base accelerations can be excited which is the case for uniaxial shaking tables. Here, if cross talk is negligible, all the rotational accelerations are zero and the moments of inertia are therefore not observable. Thus (8) can be reduced to:

$$\underbrace{\begin{bmatrix} m\ddot{\xi}^A & 0 & 0 & 0 \\ m\ddot{\eta}^A & 0 & 0 & 0 \\ m\ddot{\zeta}^A & 0 & 0 & 0 \\ 0 & 0 & m\ddot{\zeta}^A & -m\ddot{\eta}^A \\ 0 & -m\ddot{\zeta}^A & 0 & m\ddot{\xi}^A \\ 0 & m\ddot{\eta}^A & -m\ddot{\xi}^A & 0 \end{bmatrix}}_{\tilde{\mathbf{B}}} \underbrace{\begin{bmatrix} \Delta \\ \xi^S \\ \eta^S \\ \zeta^S \end{bmatrix}}_{\tilde{\sigma}} = \underbrace{\begin{bmatrix} f^A_\xi \\ f^A_\eta \\ f^A_\zeta \\ t^A_\xi \\ t^A_\eta \\ t^A_\zeta \end{bmatrix}}_{\tilde{\mathbf{f}}^A} \tag{11}$$

It can be seen that only the indicator (or the overall mass) and the location of the center of gravity can be estimated.

5 THE IDENTIFICATION ALGORITHM

The identification itself is done by solving equation (9) according to equation (10) non-iteratively for a given reference point A or iteratively. The iterative procedure uses the location of the center of gravity identified in the actual iteration step as the reference point for the next iteration step and thus yields the parameters with respect to the center of gravity after convergence is achieved. It will be shown that the systematic error introduced by the

influence of the elastic modes may be reduced in most cases due to iteration.

Input data for the identification algorithm are measured rigid body accelerations and the corresponding excitation forces, the coordinates of a (starting) reference point and the coordinates of the measurement locations. The coordinates may reference any global Cartesian coordinate system of choice. Furthermore weighting matrices can be supplied for each excitation configuration to individually emphasize the measured rigid body accelerations.

At first the needed components of the rigid body mode matrix are calculated for the actual reference point. Then the measured rigid body accelerations and excitation forces are transformed according to equations (4) and (5). After assembling the measurement matrices and the force vectors for all excitation configurations the hyper matrix system (9) is formed and solved according to equation (10). The iteration is performed by repeating the procedure until the identified location of the center of gravity coincides with the actual reference point within a chosen tolerance radius. An overview of the identification algorithm is given in figure 4.

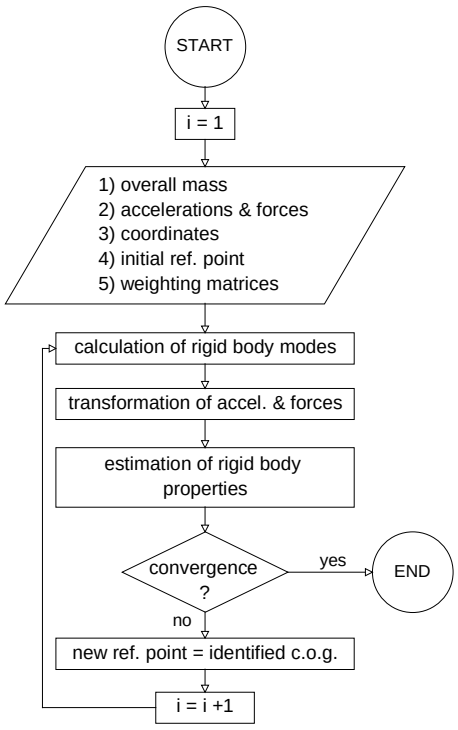

Figure 4: Overview of identification algorithm

6 HOW TO DETERMINE THE RIGID BODY ACCELERATIONS AND EXCITATION FORCES

Transformation of equation (1) into the frequency domain via:

$$\mathbf{a}^A(t) = \mathbf{A}^A(j\omega)\, e^{j\omega t} \quad , \quad \mathbf{f}^A(t) = \mathbf{F}^A(j\omega)\, e^{j\omega t}$$

with ω - circular frequency, $j = \sqrt{-1}$ yields:

$$\mathbf{M}^A\, \mathbf{A}^A(j\omega) = \mathbf{F}^A(j\omega) \qquad (12)$$

It can be seen that (12) is completely equivalent to (1) and thus all the equations derived so far remain valid if the time domain quantities are simply exchanged by their frequency domain counterparts. This furthermore means that the identification equations (8) and (9) are fulfilled for every frequency and so, theoretically, the rigid body properties could be identified by using only one single arbitrary frequency ω_i and the corresponding acceleration and force/moment spectral values $\mathbf{A}^A(j\omega_i)$, $\mathbf{F}^A(j\omega_i)$. For real test data this is not meaningful because of two reasons:

1. noise on measurement data
2. influence of the elastic modes

The noise on measurement data can be reduced by averaging the data over an appropriate frequency range. The influence of the elastic modes may be neglected if the first natural frequency is sufficiently greater than the frequency range used for averaging. Otherwise the influence of the elastic modes must be considered.

This paper compares five different methods to determine the rigid body accelerations (also called masslines) and the corresponding excitation forces:

a) from averaging base excitation frequency responses
b) from averaging base excitation power spectra
c) from averaging frequency response functions of the free system
d) from curve fitting frequency response functions of the free system
e) from averaging synthesized frequency response functions of the rigid body

Since the real part of $\mathbf{A}^A(j\omega_i)$, $\mathbf{F}^A(j\omega_i)$ in general holds the significant information all averaging methods determine the rigid body accelerations and the corresponding excitation forces from the real part only (this is valid since (12) must be fulfilled for the real part and the imaginary part of $\mathbf{A}^A(j\omega_i)$, $\mathbf{F}^A(j\omega_i)$ independently). The frequency response functions of the free system needed for c), d) and e) can be identified from base excitation test data according to the procedure proposed in [SCHEDLINSKI] if six linear independent base excitation configurations are provided.

a) averaging base excitation frequency responses

For harmonic base excitation (e.g. stepped sine signals) the real parts of the measured frequency responses $\mathbf{A}^A(j\omega_i)$, $\mathbf{F}^A(j\omega_i)$ can directly be averaged over a given frequency range well below the first elastic natural frequency. The resulting rigid body accelerations and interface forces are supplied to the identification algorithm.

b) averaging base excitation power spectra

For transient base excitation (e.g. random signals) averaged power spectra should be used. If equation (12) is multiplied from the right hand side with the conjugate complex transpose of the force spectra:

$$\mathbf{M}^{A}\ \underbrace{\mathbf{A}^{A}(j\omega)\,\mathbf{F}^{*A}(j\omega)}_{\mathbf{G}^{A}_{AF}(j\omega)} = \underbrace{\mathbf{F}^{A}(j\omega)\,\mathbf{F}^{*A}(j\omega)}_{\mathbf{G}^{A}_{FF}(j\omega)} \tag{13}$$

it can be seen that the power spectra $\mathbf{G}^{A}_{AF}(j\omega)$, $\mathbf{G}^{A}_{FF}(j\omega)$ can be used analogous to the spectra $\mathbf{A}^{A}(j\omega_i)$, $\mathbf{F}^{A}(j\omega_i)$ as shown under a).

If the influence of the first elatic modes is rather significant a Cholesky factorization may improve the results. Here the real parts of the power spectra are averaged over a given frequency range first. Then the matrix of the averaged auto power spectra is factorized:

$$\overline{\mathrm{Re}\left\{\mathbf{G}^{A}_{FF}(j\omega)\right\}} = \mathbf{R}^{T}\mathbf{R}$$

It can be shown that this factorization is always possible if six linear independent excitation configurations have been provided. In this case the matrix of the averaged auto power spectra is positive definite.

Multiplying equation (13) with $\mathbf{R}^{-1}$ from the right hand side yields:

$$\mathbf{M}^{A}\ \underbrace{\overline{\mathrm{Re}\left\{\mathbf{G}^{A}_{AF}(j\omega)\right\}}\,\mathbf{R}^{-1}}_{\mathbf{Q}} = \underbrace{\overline{\mathrm{Re}\left\{\mathbf{G}^{A}_{FF}(j\omega)\right\}}\,\mathbf{R}^{-1}}_{\mathbf{R}^{T}}$$

The pseudo accelerations and forces in $\mathbf{Q}$ and $\mathbf{R}^{T}$ are supplied to the identification algorithm.

c) averaging frequency response functions of the free system

The real parts of the frequency response functions are averaged over a given frequency range well below the first elastic natural frequency. Since the frequency response functions have been estimated with respect to unit forces the interface forces are equal to one. The averaged acceleration responses and the unity forces are supplied to the identification algorithm.

d) curve fitting frequency response functions of the free system

A special curve fitting procedure [LINK-4] can also be used to extract the rigid body response from the frequency response function data. Here the rigid body accelerations are identified as residuals in addition to the first elastic modes. These residuals and the unity forces are supplied to the identification algorithm.

e) averaging synthesized frequency response functions of the rigid body

After an experimental modal analysis of the free system the modal data extracted may be used to generate analytical frequency response functions that can be subtracted from the estimated ones. The remaining frequency response is, in the ideal case, the response of the rigid body. This synthesized response can be treated as shown under c).

7 A WEIGHTING STRATEGY

Usually the weighting matrix $\mathbf{W}$ in (4) is set equal to the unity matrix which reduces the weighted least squares approach to a standard least squares approach. In this paper, however, a weighting strategy shall be proposed that is based on the data also used to determine the rigid body accelerations. It is capable of taking the effects of the first elastic modes into account and can help to reduce their influence on the estimated rigid body accelerations at the reference point.

In order to set up the weighting matrix the response which is used to determine the rigid body accelerations is investigated in the vicinity of the first modes. Here the maximum values of the amplitudes are collected for each measured acceleration. Now if a given maximum amplitude value is large in comparison to the complete set of amplitude values the influence of the first modes on the lower frequency range and therefore on the corresponding rigid body acceleration will be relatively large and vice versa. The collected amplitude values are subsequently scaled and inverted such that the largest amplitude value arrives at a minimum weighting value w_{min} with $0 \le w_{min} < 1$ and the smallest amplitude value arrives at a maximum weighting value w_{max} equal to one (see figure 5 for an example $0.25 \le w_i \le 1$).

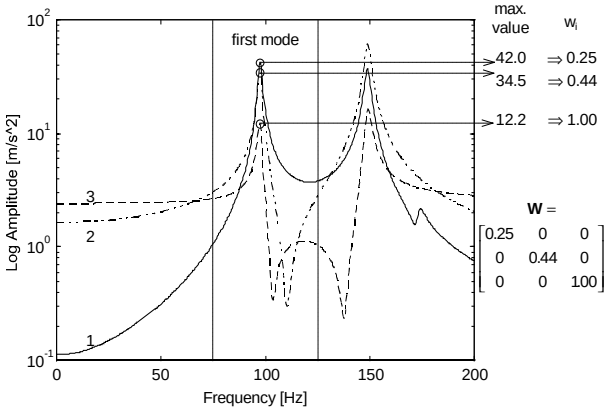

Figure 5: How to determine weighting values

The diagonal of the weighting matrix is formed from the weighting values w_i and emphasizes those rigid body accelerations with a relatively small systematic error due to the presence of the first elastic modes and thus improves the estimation result for the rigid body accelerations at the interface.

8 NUMERIC EXAMPLE

In order to get a first insight into the capabilities of the approach the 2D model of a steel tuning fork (figure 6) shall be investigated numerically. The first natural frequencies of the system are listed in tables 1 and 2. It can be seen that the system cannot be regarded as a rigid body. An influence of the first elastic modes thus has to be expected.

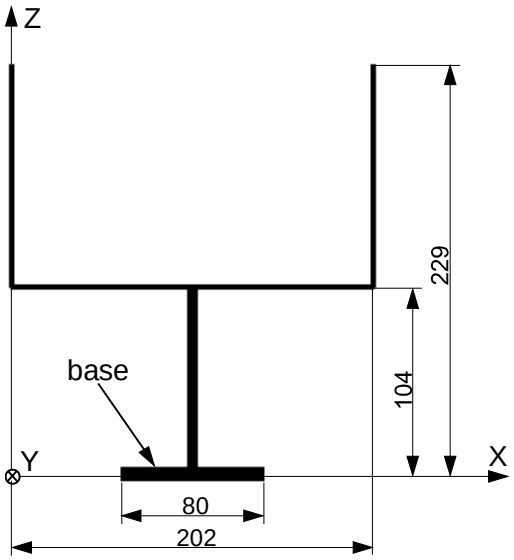

Figure 6: Survey of 2D tuning fork

#	1	2	3	4	5
f [Hz]	32.52	41.08	114.27	126.05	215.22

Table 1: Natural frequencies of fixed system

#	1-3	4	5	6	7
f [Hz]	0	43.17	97.45	148.93	173.17

Table 2: Natural frequencies of free system

The numerical test data (15 accelerations, three interface forces/moments) were produced without noise from three linear independent excitation patterns: lateral base acceleration in X and Z direction as well as rotational base acceleration about Y axis. Fourier transforms, power spectra and frequency response functions were calculated and a modal analysis of the free system was performed which provided the modal data of the free system and acceleration residuals as well. The modal data of the free system were then used to generate frequency response functions of the rigid body. Finally the rigid body accelerations and excitation forces have been determined applying the methods described in chapter 6 and the rigid body properties were estimated.

Table 3 shows the non-iterative estimation results for all methods without and with weighting ($0.25 \le w_i \le 1$) for two different averaging frequency ranges. The first averaging frequency ranges (I) goes up to about 45 %, the second (II) up to about 80 % of the first elastic natural frequency. Reference point A was the origin of the global XYZ coordinate system (see figure 6); the overall mass of 630.76 g was supplied also. The relative errors in per cent are listed in brackets.

Method	ξ^s **[mm]**	ζ^s **[mm]**	$\Theta^A_{\eta\eta}$ **[gm²]**	Δ **[%]**
ideal	101.13	107.33	20.78	100
a) I w/o W	100.93 (-0.19)	108.77 (1.34)	21.54 (3.68)	99.68
I with W	101.54 (0.41)	112.08 (4.42)	21.87 (5.26)	100.66
II w/o W	100.07 (-1.04)	114.63 (6.80)	26.18 (26.00)	98.21
II with W	103.24 (2.09)	113.64 (24.50)	26.81 (29.05)	103.51
b) I w/o W (no Cholesky)	101.04 (-0.09)	86.41 (-19.49)	132.10 (535.8)	99.82
I with W (no Cholesky)	101.06 (-0.06)	108.17 (0.77)	42.82 (106.1)	100.04
II w/o W (no Cholesky)	100.80 (-0.32)	17.68 (-83.53)	275.92 (1228)	99.63
II with W (no Cholesky)	100.54 (-0.58)	101.34 (-5.58)	126.35 (508.1)	100.26
b) I w/o W (+ Cholesky)	100.93 (-0.19)	108.37 (0.97)	22.03 (6.03)	99.68
I with W (+ Cholesky)	101.15 (0.02)	107.44 (0.10)	21.88 (5.29)	100.09
II w/o W (+ Cholesky)	100.21 (-0.91)	112.01 (4.36)	33.12 (59.39)	98.18
II with W (+ Cholesky)	101.14 (0.01)	109.29 (1.82)	32.42 (56.05)	100.37
c) I w/o W	100.66 (-0.46)	105.53 (-1.68)	21.30 (2.52)	99.54
I with W	100.59 (-0.53)	105.97 (-1.27)	21.23 (2.18)	99.50
II w/o W	97.84 (-3.25)	94.27 (-12.17)	24.81 (19.40)	96.75
II with W	98.15 (-2.95)	98.91 (-7.85)	23.98 (15.41)	97.22
d) w/o W	101.38 (0.25)	108.83 (1.40)	19.98 (-3.86)	100.25
with W	101.76 (0.63)	108.71 (1.28)	20.09 (-3.31)	100.58
e) I w/o W	101.00 (-0.12)	106.92 (-0.38)	20.82 (0.22)	99.88
I with W	101.03 (-0.10)	107.02 (-0.30)	20.82 (0.20)	99.90
II w/o W	100.54 (-0.58)	105.42 (-1.78)	20.99 (1.02)	99.42
II with W	100.67 (-0.46)	105.87 (-1.36)	20.97 (0.94)	99.56

Table 3: Non-iterative estimation results

The best results for every method are shaded. It can be seen that the weighting improves the results except for method a). A reason for this may be that here the influence of the first elastic modes is less significant than the incompatibility introduced by weighting the rigid body accelerations while the corresponding

excitation forces cannot be weighted. Method b) without Cholesky factorization can only be used if the influence of the first elastic modes is negligible. With Cholesky factorization, however, the results are still quite good if a significant influence of the first elastic modes is present. The most accurate results of course come from method e) where the elastic influences are reduced in the best possible way. However, even here a small error resides due to errors in the experimental modal analysis. Method d) is not quite as good. Maybe the results could be improved if the acceleration residuals are identified taking more than just the first elastic mode into account for the curve fitting algorithm. Another important result is, that the indicator shows no significant variation at all. However, the estimation should in any case be performed in the proposed manner since the a priory knowledge of the overall mass is likely to reduce the error on the estimated center of gravity location.

In table 4 the results of the non-iterative and the iterative estimation are listed for averaging frequency range I and weighting. Since method b) without Cholesky factorization does not provide reliable results it is omitted here. Convergence was in general achieved after two to three iteration steps.

Method	ξ^s [mm]	ζ^s [mm]	$\Theta^S_{\eta\eta}$ [gm^2]
ideal	101.13	107.33	7.06
a) I with W direct	101.54 (0.41)	112.08 (4.42)	7.44 [1)] (5.41)
iterative	101.12 (0.00)	111.06 (3.47)	7.66 (8.52)
b) I with W direct (+ Cholesky)	101.15 (0.02)	107.44 (0.10)	8.14 [1)] (15.31)
iterative	101.15 (0.02)	107.44 (0.10)	8.54 (20.88)
c) I with W direct	100.59 (-0.53)	105.97 (-1.27)	7.77 [1)] (9.98)
iterative	101.10 (-0.03)	106.51 (-0.77)	7.70 (9.01)
d) with W direct	101.76 (0.63)	108.71 (1.28)	6.10 [1)] (-13.55)
iterative	101.18 (0.05)	108.08 (0.70)	6.18 (-12.41)
e) I with W direct	101.03 (-0.10)	107.02 (-0.30)	7.16 [1)] (1.39)
iterative	101.12 (0.00)	107.12 (-0.20)	7.15 (1.20)

[1)] calculated via transformation: $\Theta^S_{\eta\eta} = \Theta^A_{\eta\eta} - m \cdot (\xi_S^2 + \zeta_S^2)$

Table 4: Non-iterative vs. iterative estimation

Again the best results have been shaded. For methods a) and b) the center of gravity location is the same or improves due to iteration. The moment of inertia, however, becomes worse. For methods c), d) and e) the results improve in general with iteration. Especially method e) provides excellent results.

9 CONCLUSIONS

An approach to identify rigid body properties with respect to a given reference point was proposed using data from base excitation testing including measured interface forces. It was shown that the data can be used in different ways to extract rigid body accelerations and excitation forces which can be supplied to a non-iterative or iterative identification procedure.

The best results were achieved using the iterative procedure with rigid body accelerations and excitation forces coming from synthesized rigid body frequency response functions when a weighting strategy based on the measured response is used to estimate the rigid body accelerations at the reference point.

Because of the promising results of the numeric study it is planned to check the procedure with noise polluted analytical data as well as with real test data in the next future.

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