ON THE BENEFITS OF UTILIZING MEASURED INTERFACE FORCES FOR PHYSICAL AND MODAL PARAMETER IDENTIFICATION

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Abstract. In this paper the methods and results of estimating the physical and modal parameters of a liquid propellant tank from base excitation on a six axes shaking table are presented. The measurement of interface forces allows not only the identification of two sets of modal data: one for the fixed/free and the other for the free/free system, but also the identification of all rigid body properties. The high consistency of the two identified parameter sets and the good correlation with analysis results show the benefits of the additional measurement of interface forces.

Key Words: Base Excitation, Identification, Interface Forces, Rigid Body Properties, Six Axes Shaking Table

1. Introduction

This paper presents the methods and an application of the physical and modal parameter identification techniques introduced in [7], [8], [9] and [10], where forces measured between the interface of the test specimen and the shaking table are used in addition to acceleration response data obtained from a vibration test on a six axes shaking table. If the interface forces are not measured only the natural frequencies, modal damping values and mode shapes of the fixed/free system can be identified using standard experimental modal analysis (EMA) techniques. However, if the interface forces are measured in addition, the following possibilities for physical and modal parameter identification arise:

• Modal masses, effective masses and mass participation factors of the fixed/free system

- in addition to the conventional modal data (natural frequencies, mode shapes and modal damping values).
- Rigid body properties (overall mass, center of gravity location and mass moments of inertia).
- Modal data of the free/free system either from frequency response functions (FRF) estimated with respect to unit interface forces or from an experimental Craig/Bampton (C/B) model of the free/free system assembled from the identified fixed/free modal data and the identified rigid body properties.

In order to take full advantage of these identification possibilities, test data from six axes shaking table testing is needed. In this paper six axes shaking table test data acquired by the Deutsche Forschungsanstalt für Luft- und Raumfahrt (DLR) in Göttingen, Germany is evaluated. The tested system was a liquid propellant tank (LPT) plus support structure mounted on a force measurement device (FMD), both provided by ESA/ESTEC, Noordwijk, The Netherlands (see figure 1).

It will be shown that the identification results correlate very well with Finite Element Analysis results. Furthermore the identification results, especially the results for the free/free modal data coming from two independent identification procedures, are very consistent and thus enhance the confidence in the accuracy of the identified quantities.

It is concluded that the additional measurement of interface forces during shaking table testing may significantly enhance the identification possibilities and underlines the importance of the used methods for modal testing and analytical model verification.

2. Theoretic Background

If the system to be investigated is mounted in a statically determinate way on a six axes shaking table and if the interface accelerations and forces are measured in addition to acceleration responses of the system, FRFs of the free/free and the fixed/free system may be identified. Based upon these FRFs physical and modal parameters can be identified subsequently [8].

The FRFs are identified using the well known equations for multiple input / multiple output systems (1) by post multiplying with the inverse auto spectral matrix $\mathbf{G}_{\mathrm{ff}}^{-1}(j\omega)$.

$$\mathbf{G}_{\mathrm{af}}(\mathrm{j}\omega) = \mathbf{H}(\mathrm{j}\omega) \ \mathbf{G}_{\mathrm{ff}}(\mathrm{j}\omega) \tag{1}$$

 $G_{af}(j\omega)$ cross spectral matrix of output (a) and input (f) signals

 $H(j\omega)$ FRF matrix

 $G_{\rm ff}(j\omega)$ auto spectral matrix of input (f) signals

In the case of n_e input signals the auto spectral matrix may be inverted if $n_v \ge n_e$ frames of test data are used to calculate averaged spectral matrices according to equations (2).

$$\overline{\mathbf{G}_{af}(j\omega)} = \frac{1}{n_{v}} \sum_{k=1}^{n_{v}} \mathbf{G}_{af}(j\omega)_{k} , \overline{\mathbf{G}_{ff}(j\omega)} = \frac{1}{n_{v}} \sum_{k=1}^{n_{v}} \mathbf{G}_{ff}(j\omega)_{k} (2)$$

In order to meet the requirements for the inversion of $\mathbf{G}_{ff}(j\omega)$ either stochastic or deterministic input can be used. In the first case the input signals must be uncorrelated, in the second n_e linear independent excitation patterns must be provided.

Using base excitation on a six axes shaking table as outlined above, FRFs of the free/free system may be identified if the interface forces (three forces and three moments: $n_e = 6$) are used as inputs (i. e. the FRFs are responses normalized to unit interface forces). If the interface accelerations (three translational and three rotational accelerations: $n_e = 6$) are interpreted as inputs and inserted into equations

(1) and (2) it can be shown that the resulting FRFs provide the information to identify the modal parameters of the fixed/free system except for the modal masses (in this case the FRFs represent responses normalized to unit interface accelerations, not to unit forces, [8]). The corresponding averaged auto spectral matrices can be inverted, for instance, if the shaking table is driven in each of its six axes separately in order to assemble $n_v \ge 6$ frames of test data.

The complete set of modal parameters of the free/free system can now be identified either directly from the free/free FRFs by standard EMA techniques (see e. g. [4], [5]) or from an experimental C/B model [2] of the free/free system assembled from identified modal data of the fixed/free system and an estimate of the rigid body properties [1].

The modal parameters of the fixed/free system except for the modal masses can also be identified by standard EMA techniques. The missing modal masses plus effective masses and mass participation factors can be extracted as well using a special EMA procedure presented in [7]. This procedure identifies the modal system matrices in modal space utilizing already identified natural frequencies and mode shapes of the fixed/free system in addition to measured interface forces.

The physical rigid body mass matrix can be identified using a two-step procedure first introduced by the authors in [9]. In a first step the underlying rigid body response is extracted from the real parts of the free/free FRFs which hold this information. Since the real part of a FRF is an even function the following biquadratic approach can be used in the frequency range below the first elastic natural frequency for each measured degree of freedom (dof) $k = 1, ..., n_m$:

$$H_k^{re}(\omega) = C_0 + C_2 \omega^2 + C_4 \omega^4$$
 (3)

Assembling data at $i = 1,..., n \ge 3$ spectral lines ω_i yields:

$$\begin{bmatrix} H_{k}^{\text{re}}(j\omega_{1}) \\ H_{k}^{\text{re}}(j\omega_{2}) \\ \vdots \\ H_{k}^{\text{re}}(j\omega_{n}) \end{bmatrix} = \begin{bmatrix} 1 \omega_{1}^{2} & \omega_{1}^{4} \\ 1 \omega_{2}^{2} & \omega_{2}^{4} \\ \vdots & \vdots & \vdots \\ 1 \omega_{n}^{2} & \omega_{n}^{4} \end{bmatrix} \begin{bmatrix} C_{0k} \\ C_{2k} \\ C_{4k} \end{bmatrix}$$

$$(4)$$

Equation (4) can now be solved in a least squares sense for $[C_{0k} C_{2k} C_{4k}]^T$ and the constant term C_{0k} then represents an estimate of the rigid body response at dof k. The estimates for all measured dof are finally assembled in the vector $\mathbf{a}^{M,T} = [C_{01},...,C_{0n_m}]$.

In a second step an iterative estimation technique based upon the linearized equations of motion of an unrestrained rigid body (5) is used in order to identify all 10 rigid body parameters contained in the matrix $\mathbf{M}_{\mathsf{R}}^{\mathsf{A}}$.

$$\begin{bmatrix} m & 0 & 0 & 0 & m\zeta^{S} - m\eta^{S} \\ 0 & m & 0 & -m\zeta^{S} & 0 & m\xi^{S} \\ 0 & 0 & m & m\eta^{S} - m\xi^{S} & 0 \\ 0 & -m\zeta^{S} & m\eta^{S} & \Theta_{\xi\xi}^{A} & -\Theta_{\xi\eta}^{A} & -\Theta_{\eta\zeta}^{A} \\ m\zeta^{S} & 0 & -m\xi^{S} - \Theta_{\xi\eta}^{A} & \Theta_{\eta\eta}^{A} & -\Theta_{\eta\zeta}^{A} \\ -m\eta^{S} & m\xi^{S} & 0 & -\Theta_{\xi\zeta}^{A} & -\Theta_{\eta\zeta}^{A} & \Theta_{\zeta\zeta}^{A} \end{bmatrix} \underbrace{ \begin{bmatrix} \ddot{u}^{A} \\ \ddot{v}^{A} \\ \ddot{w}^{A} \\ \ddot{\alpha}^{A} \\ \ddot{\beta}^{A} \\ \ddot{\gamma}^{A} \end{bmatrix}}_{\boldsymbol{A}^{A}} = \underbrace{ \begin{bmatrix} f_{\xi}^{A} \\ f_{\eta}^{A} \\ \xi^{A} \\ \vdots \\ f_{\eta}^{A} \\ t_{\zeta}^{A} \end{bmatrix}}_{\boldsymbol{A}^{A}} (5)$$

 $\begin{array}{ll} m & \text{overall mass} \\ \xi^S, \eta^S, \zeta^S & \text{center of gravity (cog) location} \\ \Theta^A_{\xi\xi}, ... & \text{mass moments of inertia w.r.t. point A} \\ u^A, \alpha^A, ... & \text{translations/rotations w.r.t. point A} \\ f_\xi^A, t_\xi^A, ... & \text{forces/moments w.r.t. point A} \end{array}$

Theoretically equation (5) could directly be used to identify the rigid body properties. Yet to reduce the number of unknowns an estimation vector σ according to equation (6) can be defined.

$$\sigma = \left[m \, m \xi^{S} \, m \eta^{S} \, m \zeta^{S} \, \Theta^{A}_{\xi \xi} \, \Theta^{A}_{\eta \eta} \, \Theta^{A}_{\zeta \zeta} \, \Theta^{A}_{\xi \eta} \, \Theta^{A}_{\xi \zeta} \, \Theta^{A}_{\eta \zeta} \right]^{T} (6)$$

Reassembling equation (5) yields equation (7) which can be solved for the 10 rigid body properties contained in σ in a least squares sense. Since equation (7) only supplies six equations for 10 unknowns test data of at least two tests with different excitation patterns must be processed simultaneously (additional data is added by appending lines to the measurement matrix **B** and the force vector \mathbf{f}^{A}).

$$\begin{bmatrix} \ddot{u}^{A} & 0 & -\ddot{\gamma}^{A} & \ddot{\beta}^{A} & 0 & 0 & 0 & 0 & 0 & 0 \\ \ddot{v}^{A} & \ddot{\gamma}^{A} & 0 & -\ddot{\alpha}^{A} & 0 & 0 & 0 & 0 & 0 & 0 \\ \ddot{w}^{A} - \ddot{\beta}^{A} & \ddot{\alpha}^{A} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \ddot{w}^{A} - \ddot{v}^{A} & \ddot{\alpha}^{A} & 0 & 0 - \ddot{\beta}^{A} - \ddot{\gamma}^{A} & 0 \\ 0 - \ddot{w}^{A} & 0 & \ddot{u}^{A} & 0 & \ddot{\beta}^{A} & 0 - \ddot{\alpha}^{A} & 0 - \ddot{\gamma}^{A} \\ 0 & \ddot{v}^{A} - \ddot{u}^{A} & 0 & 0 & 0 & \ddot{\gamma}^{A} & 0 - \ddot{\alpha}^{A} - \ddot{\beta}^{A} \end{bmatrix} \sigma = \mathbf{f}^{A}(7)$$

It is in general impossible to measure the rigid body responses \mathbf{a}^A and the corresponding forces \mathbf{f}^A directly at an arbitrary reference point A. Thus \mathbf{a}^A is estimated from the relation $\mathbf{a}^M = \mathbf{X}_R \, \mathbf{a}^A$, where \mathbf{a}^M represents the rigid body responses extracted from equation (4), via the least squares solution:

$$\mathbf{a}^{\mathsf{A}} = \left(\mathbf{X}_{\mathsf{R}}^{\mathsf{T}} \mathbf{W} \mathbf{X}_{\mathsf{R}}\right)^{-1} \mathbf{X}_{\mathsf{R}}^{\mathsf{T}} \mathbf{W} \mathbf{a}^{\mathsf{M}} \tag{8-a}$$

X_R matrix of rigid body modes at measured dofW optional weighting matrix

For a single point P, for example, \mathbf{X}_R takes the form shown in equation (9) if all translational accelerations are measured.

$$\mathbf{X}_{R} = \begin{bmatrix} 1 & 0 & 0 & 0 & \zeta^{P} & -\eta^{P} \\ 0 & 1 & 0 & -\zeta^{P} & 0 & \xi^{P} \\ 0 & 0 & 1 & \eta^{P} & -\xi^{P} & 0 \end{bmatrix}$$
(9)

 ξ^P , η^P , ζ^P coordinates of point P w.r.t. point A At last, the forces at the reference point A \mathbf{f}^A follow from the transformation:

$$\mathbf{f}^{\mathsf{A}} = \left(\mathbf{X}_{\mathsf{R}}^{\mathsf{FMD}}\right)^{\mathsf{T}} \mathbf{f}^{\mathsf{FMD}} \tag{8-b}$$

 $\mathbf{X}_{R}^{\text{FMD}}$ matrix of rigid body modes, w.r.t. FMD coordinates \mathbf{f}^{FMD} vector of measured FMD forces

A more detailed discussion of the technique is given in references [9] and [10].

3. The Tested System

The tested system is a liquid propellant tank (LPT, diameter 700 mm, height 1323 mm, mass 47.7 kg) with support structure (height 1900 mm, width/length 804 mm, mass 567 kg) mounted on a special force measurement device (FMD, assembly of two steel rings connected by eight piezoelectric force transducers, mean diameter 1194 mm, mass 280 kg, [12]) provided by ESA/ESTEC, Noordwijk, The Netherlands. The test data evaluated was acquired by the Deutsche Forschungsanstalt für Luft- und Raumfahrt (DLR) in Göttingen, Germany, during six tests. For each test the complete system LPT/FMD was base excited in another direction with the six axes shaking table system MAVIS 2 [3].

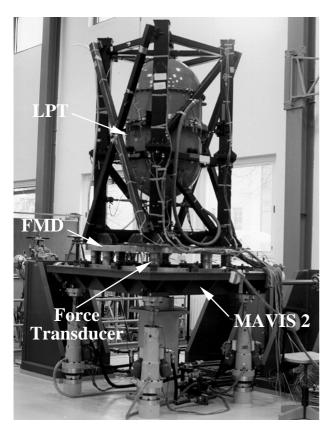


Figure 1: LPT/FMD on MAVIS 2 (DLR Göttingen)

4. Finite Element Analysis

In order to produce analytical data for correlation and test data evaluation purposes a Finite Element (FE) analysis was performed. Two models, one for the LPT and one for the FMD were provided by ESA/ESTEC. Since the upper half of the FMD (the part above of the force transducers) becomes part of the tested system the two FE models were merged (figure 2) in order to yield analysis results comparable to the identification results.

The rigid body properties for this model are:

Property	Value
cog w.r.t. origin	$[0,00\ 0,00\ 0,51]^{\mathrm{T}}$ m
overall mass	920,40 kg
Principal moment of inertia (moi) 1	264,80 kgm²
Principal moment of inertia (moi) 2	513,00 kgm²
Principal moment of inertia (moi) 3	514,50 kgm²

Table 1: Analytical rigid body properties w.r.t. cog

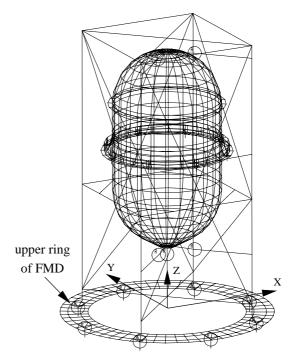


Figure 2: Merged FE model of LPT/FMD

An analytical modal analysis in the frequency range from zero to 150 Hz yielded:

#	Frequency [Hz]	Mode Description			
1	71.60	1. bending (X)			
2	75.56	1. bending (Y)			
3	86.73	1. torsion			
4	93.29	crossbars (sym.)			
5	98.96	crossbars (anti.)			
6	120.56	2. torsion			
7	135.91	2. bending (X)			
8	141.54	tank vertical			

Table 2: FE analysis results - fixed/free system

#	Frequency [Hz]	Mode Description			
1-6	0.00	rigid body motion			
7	93.29	crossbars (sym.)			
8	96.18	crossbars (anti.)			
9	101.49	1. torsion			
10	128.94	2. torsion			
11	141.53	1. bending (X)			
12	146.78	tank vertical			

Table 3: FE analysis results - free/free system

5. The Test

The test data were acquired during six sweep sine tests covering a frequency range from 10 to 120 Hz. For each test the complete system LPT/FMD was base excited in another direction (three translational and three rotational table movements, [11]). The fourier transforms of the measured forces and accelerations were calculated at DLR. For each of the six test runs the following data were acquired:

- six interface forces/moments
- six interface accelerations
- 78 acceleration responses (see figure 3)

leading to six independent sets of data to be used for physical and modal identification.

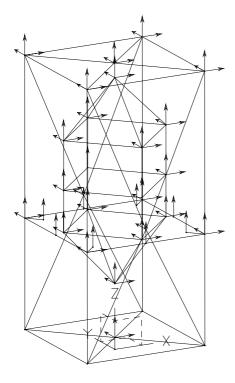


Figure 3: Measured acceleration responses

6. Identification Results

a) FRF

At first, FRFs of the free/free and the fixed/free system were identified. Typical sets of FRFs for selected dof (upper part of the support frame and crossbars) are shown in figure 4 and figure 5.

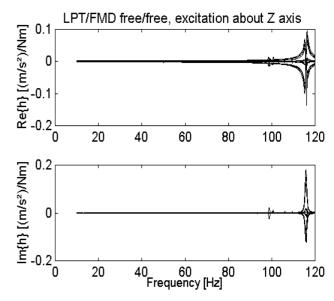


Figure 4: Selected FRFs of the free/free system

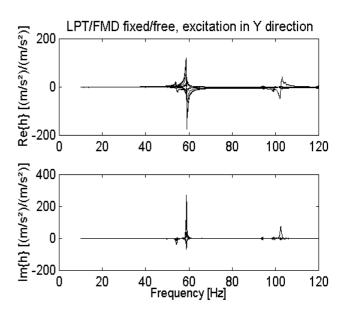


Figure 5: Selected FRFs of the fixed/free system

a) Free/Free System From EMA

The modal identification of the free/free system from free/free FRFs was performed using a standard EMA technique [6]. The results are listed in table 4. The correlation shows that all three modes found in the analysis could also be identified. Even the modal masses agree well except for the first mode. Here the identified value seems to be to high. Since the first two modes are both local modes of the crossbars one would expect that the values of the modal masses are nearly the same for these modes. The modal damping values indicate that the system is extremely lightly damped. This increases the

uncertainty of the identified modal mass values, because only a limited amount of significant spectral information (around the response peaks) can be used for EMA.

_	uency [z]	MAC [%]	Modal Mass [kg]		Damping [%]
Test	FEA		Test FEA		
98.78	93.29	99.5	56.31	18.26	0.25
	(-5.56)			(-67.57)	
100.52	96.18	98.6	16.46	18.53	0.08
	(-4.32)		(12.58)		
115.77	101.49	90.0	107.94 78.11		0.39
	(-12.33)			(-27.64)	

Table 4: Identification results, free/free system (values in brackets: relative deviation in percent)

An example of the identified mode shapes can be found in figure 6.

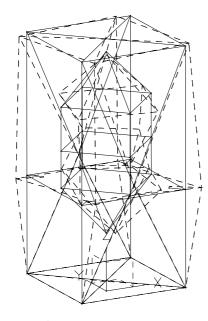


Figure 6: Free/free system - 115.77 Hz

b) Identification Of Rigid Body Properties

The rigid body properties were identified using the two-step procedure outlined above.

It can be seen that the overall mass and the center of gravity location (cog) differ less than 10 percent from the analytical results. The principal moments of inertia (moi), however, differ more. In order to better evaluate the quality of the identified values a comparison with values determined by different identification techniques (weighing, pendulum testing) would be helpful. Yet these values were not available.

Parameter	Test	FEA
overall mass [kg]	836.00	920.40 (10.10)
cog, x [m]	0.01	0.00 (-)
cog, y [m]	0.01	0.00 (-)
cog, z [m]	0.47	0.51 (8.51)
moi 1 [kgm²]	228.73	264.80 (15.77)
moi 2 [kgm²]	365.95	513.00 (40.18)
moi 3 [kgm²]	394.54	514.50 (30.41)

Table 5: Identified rigid body properties w.r.t. cog (values in brackets: relative deviation in percent)

c) Fixed/Free System From EMA

A standard EMA technique [6] was used in order to identify natural frequencies, mode shapes and modal damping degrees of the fixed/free system from fixed/free FRFs. The modal masses, effective masses and mass participation factors were extracted using the special EMA technique [7]. The results are shown in table 6.

_	Frequency [Hz]		Modal Mass [kg]		Damping [%]
Test	FEA		Test	FEA	
54,13	71,60 (32,27)	87,9	158,18	222,57 (40,71)	0,51
58,88	75,56 (28,34)	88,2	117,81	270,36 (129,49)	0,33
75,60	-	- 1)	-	-	0,45
94,17	86,73 (-7,91)	92,8	168,52	135,75 (-19,45)	0,27
98,86	93,29 (-5,63)	99,5	22,89	18,02 (-21,28)	0,29
102,52	98,96 (-3,47)	92,0	19,73	23,13 (-17,23)	0,38
115,26	135,91 (17,92)	58,8 ²⁾	-	-	-
116,28	-	- 1)	-	-	-
118,92	141,54 (19,02)	84,2	-	-	-

¹⁾ no pairing possible / ²⁾ pairing by visual inspection but MAC < 80 % *Table 6: Identification results, fixed/free system* (values in brackets: relative deviation in percent)

The correlation shows that only the sixth analytical mode shape could not be identified.

This mode probably lies outside the frequency range covered by the test. Moreover two additional modes were identified that could not be found in the analysis.

For the third mode no modal mass, for the seventh, eight and ninth mode neither modal mass nor modal damping could be identified. The modal masses of the fourth, fifth and sixth mode correlate well with the analytical results. For the first two modes, however, the deviations are larger. The large deviations between identified and calculated natural frequencies and modal masses may be traced back to modeling uncertainties of the connections between the LPT and the FMD as well as between the FMD and the shaking table. Again the modal damping values indicate that the system is very lightly damped increasing the uncertainty of the identified modal mass values.

Table 7 lists the results for the effective masses.

#	X-trans. [kg]		Y-tran	ıs. [kg]	Z-tran	s. [kg]
	Test	FEA	Test	FEA	Test	FEA
1	318,62	390,01 (22,41)	32,75	0,00	0,38	0,00
2	30,70	0,00	337,89	349,01 (3,29)	0,13	0,02
31)	-	-	-	-	-	-
4	0,42	0,24	0,06	0,00	0,59	0,00
5	0,00	0,00	0,07	0,00	0,78	0,01
6	0,02	0,00	3,36	13,63	0,00	0,01

#	X-rot. [kg]		Y-rot	. [kg]	Z-rot	. [kg]
	Test	FEA	Test	FEA	Test	FEA
1	49.77	0.00	480.20	639.45 (33,16)	0.01	0.09
2	593.91	596.13 (0,37)	45.45	0.00	0.00	0.01
31)	-	-	-	-	-	-
4	0.07	0.00	0.00	1.29	89.08	83.65 (-6,10)
5	0.06	0.00	0.00	0.00	0.27	0.00
6	9.58	58.86	0.06	0.00	0.05	0.01

¹⁾ no pairing possible

Table 7: Effective masses (values in brackets: relative deviation in percent)

The large values of the effective masses of the first, second and fourth mode correlate well with the analysis results. However, the values for the fifth and sixth mode correlate less. This may be explained by the fact that these modes are local modes of the crossbars with small effective mass values.

As an example for the identified mode shapes the first bending mode is shown in figure 7.

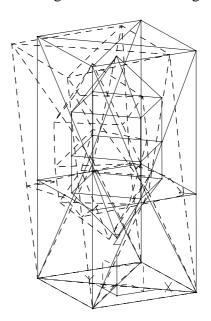


Figure 7: Fixed/free system - 54.13 Hz

d) Free/Free System From C/B Model

The modal parameters of the free/free system were alternatively identified using an experimental C/B model of the free/free system. Proceeding this way yielded the same three modes as the EMA plus two computational modes which could be separated easily because of their appearance and natural frequency. The results are listed in table 8.

_	uency [z]	MAC [%]	Modal Mass [kg]		Damping [%	
EMA	C/B		EMA	C/B	EMA	C/B
98,78	98,88 (0,10)	99,0	56,31	21,10 (-62,53)	0,25	0,29 (16,00)
100,52	101,35 (0,83)	95,6	16,46	17,67 (7,35)	0,08	0,38
115,77	120,08 (3,72)	94,9	107,94	170,39 (57,86)	0,39	0,35 (-10,26)

Table 8: Free/free system from C/B model (values in brackets: relative deviation in percent)

The results of the EMA and the ones from the Craig/Bampton model agree very well, thus increasing the confidence in the identified parameters. Merely the modal masses of the first and the third mode as well as the modal damping degrees of the second mode do not correlate in a satisfying manner. These parameters should therefore be considered with lower confidence than the others.

7. Conclusions

In this paper it was shown that the additional measurement of interface forces during base excitation testing significantly enhances the identification possibilities, since the complete set of modal data of the fixed/free and the free/free system can be identified as well as the rigid body properties. These additional data can be used e. g. for model updating purposes in order to achieve a more unique solution for the uncertain system parameters.

Since the investigated LPT/FMD system is a 'real life' application and not a simple laboratory test case the practical importance of the methods is underlined. Moreover, the consistency of the identified data, especially the consistency of the free/free modal data coming from two independent identification procedures, enhances the confidence in the accuracy of the identified quantities.

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