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IDENTIFICATION OF FREQUENCY RESPONSE FUNCTIONS AND MODAL DATA FROM BASE EXCITATION TESTS USING MEASURED INTERFACE FORCES

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ABSTRACT

Base excitation testing is used in industry in order to qualify mechanical systems with respect to specified base acceleration levels. This type of excitation only allows to identify eigenfrequencies, mode shapes and modal damping values of the fixed/free system. Modal masses, mass participation factors and effective masses of the fixed/free system as well as the modal data of the free/free system cannot be identified because the excitation forces are unknown.

This paper introduces an approach to identify these modal data as well. For this purpose the reaction forces at the table/structure interface have to be measured also. Furthermore, the verification of the theory using a laboratory test structure will be presented.

NOMENCLATURE

CFRP	Carbon Fiber Reinforced Plastics
dof	degree of freedom
FE, FEA	Finite Element ..., Finite Element Analysis
FMD	Force Measurement Device
FRF	Frequency Response Function
MAC	Modal Assurance Criteria
TSP	Truss Supported Platform (test structure)
b	‘base’ or ‘’interface’
dyn	‘dynamic’
eff	‘effective’
G	auto/cross spectra matrix
H	FRF matrix
M, **D**, **K**	mass, damping, stiffness matrices
rel	‘relative’
s	‘slave’ or ‘structural’
u, **f** / **U**, **F**	displacement, force vectors/matrices
ω	circular eigenfrequency
Y	modal matrix

INTRODUCTION

Base excitation testing on a shaking table is frequently used in industry to qualify mechanical systems with respect to specified base acceleration levels. The tests are run separately for the axial and the lateral directions.

The excitation of one single axis will generally not allow to excite all the modes in the test frequency range. It would therefore be desirable to excite as many of the maximal 6 directions (3 translational, 3 rotational) as possible and to extract the fixed/free system's modal data from frequency response functions (FRFs) related to unit base accelerations. Yet even then it would not be possible to identify the modal masses, mass participation factors and effective masses of the fixed/free system because the base excitation forces are unknown. With additionally measured interface forces it would however be possible to estimate these data as well as FRFs with respect to unit reaction forces (3 translational, 3 rotational) which allow to estimate the modal data of the free/free system also.

In this paper a procedure to extract all modal data of the fixed/free system as well as of the free/free system will be presented which is based on the additional measurement of base reaction forces.

The whole procedure consists of the following steps:

1. Excitation of the system at its base in 6 independent directions by uncontrolled impacts,
2. measurement of the reaction forces with a commercially available force measurement device (FMD) in addition to measurement of structural response and base (interface) accelerations,
3. estimation of the FRFs with respect to the 6 unit base reaction forces (3 translational, 3 rotational),
4. extraction of free/free structure's modal parameters,
5. estimation of the FRFs with respect to the 6 unit base accelerations (3 translational, 3 rotational),

6. extraction of fixed/free structure's modal parameters including effective masses.

This procedure was verified using a laboratory test structure representative for the structure of an aerospace component. Correlation with analytical predictions will also be presented in the paper.

BASIC EQUATIONS FOR FRF ESTIMATION

A discrete, linear, time invariant mechanical system with n degrees of freedom (dof) can be described by equation (1). Note that (1) in general describes a free system with no dofs restrained.

$$\mathbf{M}\,\ddot{\mathbf{u}}(t) + \mathbf{D}\,\dot{\mathbf{u}}(t) + \mathbf{K}\,\mathbf{u}(t) = \mathbf{f}(t) \tag{1}$$

with **M**, **D**, **K** constant mass, damping and stiffness matrices, (n,n)
u(t), **f**(t) displacement and force vectors (functions of time), (n,1)

The equation of motion (1) described in the frequency domain is given by

$$\mathbf{K}^{dyn}(j\omega)\,\mathbf{u}(j\omega) = \mathbf{f}(j\omega) \tag{2}$$

where the (n,n) dynamic stiffness matrix of the system is defined by

$$\mathbf{K}^{dyn}(j\omega) = (-\omega^2\,\mathbf{M} + j\omega\,\mathbf{D} + \mathbf{K}) \tag{3}$$

The inversion of this equation yields the frequency response

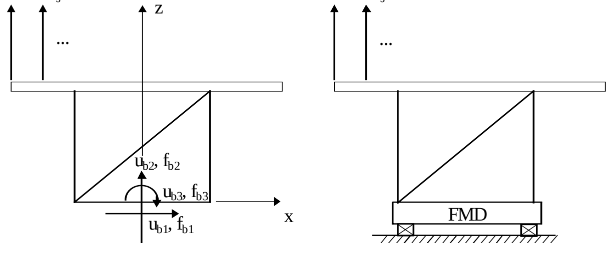
$$\mathbf{u}(j\omega) = \mathbf{H}(j\omega)\,\mathbf{f}(j\omega)$$
$$\text{with}\quad \mathbf{H}(j\omega) = \left[\mathbf{K}^{dyn}(j\omega)\right]^{-1},\ \text{frequency response function (FRF) matrix} \tag{4}$$

Equations (2) and (4) are the basis for deriving the estimation techniques described below which are themselves obtained from common FRF estimation methods presented for instance in the books of Ewins (1986) and Natke (1992).

Estimation of FRFs of the Free/Free System. Separation of Interface dof.

In order to estimate FRFs of the free/free system we now focus on (4) and consider the free/free system to be restrained at n_b given base interface dof (see figure 1 below).

Figure 1: Mechanical System, left: not restrained,
right: restraint at dof u_{b1} to u_{b3} (mounted on FMD)



Equation (4) can then be partitioned as follows

$$\begin{bmatrix}\mathbf{u}_b(j\omega)\\ \mathbf{u}_s(j\omega)\end{bmatrix} = \begin{bmatrix}\mathbf{H}_{bb}(j\omega) & \mathbf{H}_{bs}(j\omega)\\ \mathbf{H}_{sb}(j\omega) & \mathbf{H}_{ss}(j\omega)\end{bmatrix}\begin{bmatrix}\mathbf{f}_b(j\omega)\\ \mathbf{f}_s(j\omega)\end{bmatrix} \tag{5}$$

with $\mathbf{u}_b(j\omega)$, $\mathbf{f}_b(j\omega)$ base dof related quantities, $(n_b,1)$
$\mathbf{u}_s(j\omega)$, $\mathbf{f}_s(j\omega)$ slave dof related quantities, $(n-n_b,1)$

Special Case: Base Excitation.

For the special case of base excitation the forces at the slave dofs are equal to zero and only the n_b interface forces $\mathbf{f}_b(j\omega)$ are non zero. Thus (5) reduces to

$$\begin{bmatrix}\mathbf{u}_b(j\omega)\\ \mathbf{u}_s(j\omega)\end{bmatrix} = \begin{bmatrix}\mathbf{H}_{bb}(j\omega) & \mathbf{H}_{bs}(j\omega)\\ \mathbf{H}_{sb}(j\omega) & \mathbf{H}_{ss}(j\omega)\end{bmatrix}\begin{bmatrix}\mathbf{f}_b(j\omega)\\ \mathbf{0}\end{bmatrix} = \begin{bmatrix}\mathbf{H}_{bb}(j\omega)\\ \mathbf{H}_{sb}(j\omega)\end{bmatrix}\mathbf{f}_b(j\omega)$$
$$\mathbf{f}_b(j\omega) = \left[f_{b,1}(j\omega)\ldots f_{b,n_b}(j\omega)\right]^T \tag{6}$$

The FRFs in (6) contain n_b-columns of the free/free system's FRF matrix. Each of these columns can be used redundantly for identification of the free/free systems modal parameters.

Now, it can easily be seen that an estimation of FRFs directly from (6) can only be done for one-dimensional structures since in that case only one ($n_b = 1$) interface force exists. This represents the classical single point excitation case where all classical FRF estimation techniques may be applied. For two- and three-dimensional cases however, the force vector contains $n_b = 3$ respectively $n_b = 6$ non-zero components which does not allow to resolve the FRF matrix from (6) unless the number of excitation directions is increased to n_b.

Estimation of Frequency Response Functions.

For $n_b > 1$ $\mathbf{f}_b(j\omega)$ cannot be inverted because of $\mathbf{f}_b(j\omega)$ being a $(n_b,1)$ vector. If we restrict ourselves to systems that are restrained in a statically determined way three possible cases may arise:

$n_b = 1$: 1-dimensional (1D) case, e.g. single point excitation of a structure with only one non zero interface force.

$n_b = 3$: 2-dimensional (2D) case, e.g. excitation of a symmetric test structure in one plane, such that only three interface forces are non zero. (If the structure in figure 1 is symmetric to the x-z-plane only three interface forces f_{b1}, f_{b2} and f_{b3} will arise.)

$n_b = 6$: 3-dimensional (3D) case, excitation with all interface forces not equal to zero.

Providing n_b linear independent interface force vectors resulting from n_b independent base excitation directions equation (6) can be extended to

$$\begin{bmatrix}\mathbf{U}_b(j\omega)\\ \mathbf{U}_s(j\omega)\end{bmatrix} = \begin{bmatrix}\mathbf{H}_{bb}(j\omega)\\ \mathbf{H}_{sb}(j\omega)\end{bmatrix}\mathbf{F}_b(j\omega) \tag{7}$$

with
$\mathbf{U}_b(j\omega) = \left[\mathbf{u}_b^1(j\omega) \,\vdots\, \mathbf{u}_b^2(j\omega) \,\vdots\, \ldots \,\vdots\, \mathbf{u}_b^{n_b}(j\omega)\right]$ matrix of base displacements, (n_b,n_b)

$\mathbf{U}_s(j\omega) = \left[\mathbf{u}_s^1(j\omega) \vdots \mathbf{u}_s^2(j\omega) \vdots \ldots \vdots \mathbf{u}_s^{n_b}(j\omega)\right]$ matrix of slave displacements, $(n-n_b,n_b)$

$\mathbf{F}_b(j\omega) = \left[\mathbf{f}_b^1(j\omega) \vdots \mathbf{f}_b^2(j\omega) \vdots \ldots \vdots \mathbf{f}_b^{n_b}(j\omega)\right]$ matrix of interface forces, (n_b,n_b)

i.e. the six linear independent vectors are arranged as columns of a matrix. The interface force matrix is now a non singular (n_b,n_b) matrix (if the excitation directions are independent) and can thus be inverted.

Because of noise on real test data the FRF submatrix will not be calculated by inversion of the interface force matrix. It is better to use the H1 method which reduces the error on the estimated FRF submatrix in a least square sense. Here the auto- and cross spectra are built with the conjugate complex transpose interface force matrix (superscript *) and averaged over k = 1 ... r frames of measurement data (frame = data from one single excitation):

$$\sum_{k=1}^{r}\left[\begin{array}{c}\mathbf{U}_b(j\omega)\\ \hdashline \mathbf{U}_s(j\omega)\end{array}\right]_{(k)} \overset{*}{\mathbf{F}}_b(j\omega)_{(k)} = \left[\begin{array}{c}\mathbf{H}_{bb}(j\omega)\\ \hdashline \mathbf{H}_{sb}(j\omega)\end{array}\right] \sum_{k=1}^{r} \mathbf{F}_b(j\omega)_{(k)} \overset{*}{\mathbf{F}}_b(j\omega)_{(k)} \qquad (8)$$

With the averaged auto- and cross spectra matrices (9)

$$\mathbf{G}_{FF} = \sum_{k=1}^{r} \mathbf{F}_b(j\omega)_{(k)} \overset{*}{\mathbf{F}}_b(j\omega)_{(k)} \quad \text{(auto spectra matrix)}$$

$$\mathbf{G}_{UF} = \sum_{k=1}^{r}\left[\begin{array}{c}\mathbf{U}_b(j\omega)\\ \hdashline \mathbf{U}_s(j\omega)\end{array}\right]_{(k)} \overset{*}{\mathbf{F}}_b(j\omega)_{(k)} \quad \text{(cross spectra matrix)} \qquad (9)$$

equation (8) yields

$$\mathbf{G}_{UF} = \left[\begin{array}{c}\mathbf{H}_{bb}(j\omega)\\ \hdashline \mathbf{H}_{sb}(j\omega)\end{array}\right] \mathbf{G}_{FF} \qquad (10)$$

Now the averaged auto spectra matrix can be inverted and the desired estimation result for the FRF submatrix of the free system is

$$\left[\begin{array}{c}\mathbf{H}_{bb}(j\omega)\\ \hdashline \mathbf{H}_{sb}(j\omega)\end{array}\right] = \mathbf{G}_{UF}\, \mathbf{G}_{FF}^{-1} \qquad (11)$$

Estimation of FRFs of the Fixed/Free System Separation of Interface dof.

In order to estimate the FRFs of the fixed/free system with respect to unit interface displacements (or accelerations) we partition (2) in analogy to (5) which yields

$$\left[\begin{array}{c:c}\mathbf{K}_{bb}^{dyn}(j\omega) & \mathbf{K}_{bs}^{dyn}(j\omega)\\ \hdashline \mathbf{K}_{sb}^{dyn}(j\omega) & \mathbf{K}_{ss}^{dyn}(j\omega)\end{array}\right]\left[\begin{array}{c}\mathbf{u}_b(j\omega)\\ \hdashline \mathbf{u}_s(j\omega)\end{array}\right] = \left[\begin{array}{c}\mathbf{f}_b(j\omega)\\ \hdashline \mathbf{f}_s(j\omega)\end{array}\right] \qquad (12)$$

with $\mathbf{u}_b(j\omega)$, $\mathbf{f}_b(j\omega)$ interface dof related quantities, $(n_b,1)$
$\mathbf{u}_s(j\omega)$, $\mathbf{f}_s(j\omega)$ slave dof related quantities, $(n-n_b,1)$

Special Case: Base Excitation.

As already mentioned, for base excitation the forces at slave dof are equal to zero. We now develop the second row of (12) and arrive at

$$\mathbf{K}_{sb}^{dyn}(j\omega)\, \mathbf{u}_b(j\omega) + \mathbf{K}_{ss}^{dyn}(j\omega)\, \mathbf{u}_s(j\omega) = \mathbf{0} \qquad (13)$$

If we restrict ourselves again to systems that are restrained in a statically determined way we can split the slave dof displacements into two parts - one rigid body part and one relative displacement part:

$$\mathbf{u}_s(j\omega) = \mathbf{Y}_R \mathbf{u}_b(j\omega) + \mathbf{u}_s^{rel}(j\omega)\ , \qquad (14)$$

with $\mathbf{u}_b(j\omega) = \left[u_{b,x}(j\omega)\ u_{b,y}(j\omega)\ u_{b,z}(j\omega)\ u_{b,xx}(j\omega)\ u_{b,yy}(j\omega)\ u_{b,zz}(j\omega)\right]^T$

containing the base displacements and rotations.

I.e. the overall displacement can be described as a combination of a rigid body motion of the system and a dynamic relative motion added to the rigid body motion. The rigid body modes $\mathbf{Y}_R$ merely represent a geometrical transformation that transforms the interface displacements on the slave dof. Using (14) and (13) plus neglecting mass and damping coupling terms (13) yields the equation of motion of the fixed/free system

$$\mathbf{K}_{ss}^{dyn}(j\omega)\, \mathbf{u}_s^{rel}(j\omega) = \mathbf{f}_{eff}(j\omega) \qquad (15)$$

where the effective excitation force $\mathbf{f}_{eff}(j\omega)$ is given by

$$\mathbf{f}_{eff}(j\omega) = \omega^2\, \mathbf{M}_{ss} \mathbf{Y}_R \mathbf{u}_b(j\omega) \qquad (16)$$

The frequency response is described by

$$\mathbf{u}_s^{rel}(j\omega) = \tilde{\mathbf{H}}_{sb}\, \mathbf{u}_b(j\omega) \qquad (17)$$

with $\tilde{\mathbf{H}}_{sb} = \omega^2 \left[\mathbf{K}_{ss}^{dyn}(j\omega)\right]^{-1} \mathbf{M}_{ss}\, \mathbf{Y}_R$ (18)

$\tilde{\mathbf{H}}_{sb}$ represents a modified (n_s,n_b) FRF matrix which allows to identify the modal parameters of the fixed/free system.

Equation (17) is the analogy to equation (6) in the previous chapter and builds the basis for estimating parts of the FRF matrix $\tilde{\mathbf{H}}_{sb}$ from measured $\mathbf{u}_s^{rel}(j\omega)$ and $\mathbf{u}_b(j\omega)$.

Estimation of Frequency Response Functions.

In the general case of uncontrolled base motion $\mathbf{u}_b(j\omega)$ cannot be inverted because of $\mathbf{u}_b(j\omega)$ being a $(n_b,1)$ vector with $n_b > 1$. This can only be done for the special case of uniaxial base excitation, with, for example:

$$\mathbf{u}_b(j\omega) = \left[u_{b,x}(j\omega) \neq 0,\ u_{b,y}(j\omega) = u_{b,z}(j\omega) = u_{b,xx}(j\omega) = u_{b,yy}(j\omega) = u_{b,zz}(j\omega) = 0\right]^T$$

Here, the desired FRFs can be obtained directly from the measured response by classical estimation techniques based on single point excitation.

If we restrict ourselves to systems that are restrained in a statically determined way it follows that $n_b = 6$ in 3D, $n_b = 3$ in 2D and $n_b = 1$ in 1D.

Providing n_b linear independent base displacement vectors equation (17) can be extended to

$$\mathbf{U}_s^{rel}(j\omega) = \tilde{\mathbf{H}}_{sb}\ \mathbf{U}_b(j\omega) \tag{19}$$

with

$\mathbf{U}_b(j\omega) = \left[\mathbf{u}_b^1(j\omega) \vdots \mathbf{u}_b^2(j\omega) \vdots \ldots \vdots \mathbf{u}_b^{n_b}(j\omega)\right]$ matrix of base displacements, (n_b, n_b)

$\mathbf{U}_s^{rel}(j\omega) = \left[\mathbf{u}_s^{rel\,1}(j\omega) \vdots \mathbf{u}_s^{rel\,2}(j\omega) \vdots \ldots \vdots \mathbf{u}_s^{rel\,n_b}(j\omega)\right]$ matrix of relative slave displacements, $(n-n_b, n_b)$

i.e. the n_b linear independent vectors are arranged as columns of a matrix. The interface displacement matrix is now a non singular (n_b, n_b) matrix and can thus be inverted.

Again the H1 method which reduces the error on the estimated FRF submatrix in a least square sense is used. The auto- and cross spectra are built with the conjugate complex transpose interface displacement matrix (superscript *) and averaged over k = 1 ... r frames of measurement data:

$$\sum_{k=1}^{r} \mathbf{U}_{s\,(k)}^{rel}\ \overset{*}{\mathbf{U}}_b(j\omega)_{(k)} = \tilde{\mathbf{H}}_{sb} \sum_{k=1}^{r} \mathbf{U}_b(j\omega)_{(k)}\ \overset{*}{\mathbf{U}}_b(j\omega)_{(k)} \tag{20}$$

With the averaged auto- and cross spectra matrices (21)

$$\mathbf{G}_{U_bU_b} = \sum_{k=1}^{r} \mathbf{U}_b(j\omega)_{(k)}\ \overset{*}{\mathbf{U}}_b(j\omega)_{(k)} \quad \text{(auto spectra matrix)}$$

$$\mathbf{G}_{U^{rel}U_b} = \sum_{k=1}^{r} \mathbf{U}_{s\,(k)}^{rel}\ \overset{*}{\mathbf{U}}_b(j\omega)_{(k)} \quad \text{(cross spectra matrix)} \tag{21}$$

(20) becomes

$$\mathbf{G}_{U^{rel}U_b} = \tilde{\mathbf{H}}_{sb}\ \mathbf{G}_{U_bU_b} \tag{22}$$

Now the averaged auto spectra matrix can be inverted. The desired estimation result for the FRF submatrix that can be used to identify the eigenfrequencies and mode shapes (but not the modal masses) of the fixed/free system is obtained from

$$\tilde{\mathbf{H}}_{sb} = \mathbf{G}_{U^{rel}U_b}\ \mathbf{G}_{U_bU_b}^{-1} \tag{23}$$

The measured interface forces $\mathbf{F}(j\omega)$ are related to the measured base excitation matrix $\mathbf{U}_b(j\omega)$ by

$$\mathbf{F}(j\omega) = \mathbf{H}^F_{FU_b}\ \mathbf{U}_b(j\omega) \tag{24}$$

where $\mathbf{H}^F_{FU_b}$ represents the (n_b, n_b) force FRF matrix related to unit base displacements.

Replacing the measured displacements $\mathbf{U}_s^{rel}$ in equations (20) and (21) by the measured interface forces $\mathbf{F}(j\omega)$ allows the estimation of the forces due to unit base excitation

$$\mathbf{H}^F_{FU_b} = \mathbf{G}_{FU_b}\ \mathbf{G}_{U_bU_b}^{-1} \tag{25}$$

IDENTIFICATION OF MODAL PARAMETERS

With equations (11), (23) and (25) we finally assembled the information allowing an identification of the free/free system's modal parameters as well as the fixed/free system's modal parameters.

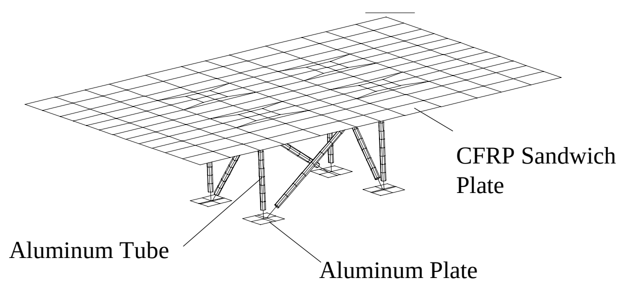
The modal parameters of the free/free system can be obtained from the FRFs estimated via (11) with standard modal extraction procedures. From the FRFs estimated via (23), however, only eigenfrequencies, mode shapes and modal damping values of the fixed/free system can be extracted. With the help of (25), though, the modal masses, mass participation factors and effective masses can be identified also using a special procedure presented by Link and Qian (1994) that will not be further discussed here.

VERIFICATION OF THE METHOD

The method was verified using a laboratory model of a space craft component called TSP (Truss Supported Platform) that has been designed and manufactured especially for modal identification and analytical model updating purposes. The main goal has been to place the most significant modes and eigenfrequencies in a frequency range from 20 to 200 Hz.

The TSP can be devided into two main parts. The platform and the support truss. The platform consists of a 800 by 1200 mm sandwich plate with aluminum honeycomb core (thickness 7.2 mm) and Carbon Fiber Reinforced Plastics (CFRP) face sheets (thickness 0.9 mm). The CFRP plate is mounted on the support truss as shown in figure 2. The struts of the truss are made of aluminum tubes with a diameter 15 mm and a wall thickness of 1 mm.

Figure 2: Survey of the TSP structure



Finite Element Analysis

The Finite Element Analysis to produce the data for test/analysis correlation has been performed with the program system IDEAS. The FE model for the free/free structure consisted of 436 nodes, 392 elements and 2616 dof, the one for the fixed/free structure of 385 nodes, 336 elements and 2246 dof. The analysis yielded eigenfrequencies and mode shapes from zero to 220 Hz. The eigenfrequencies for the free/free and the fixed/free model are presented in table 1 and table 2.

Base Excitation Test

As described above the test set-up had to provide n_b linear independent interface force vectors as well as n_b linear independent base acceleration vectors. Because of the TSP structure not being symmetric the number of possible interface forces is six (3D case) if the structure is mounted in a statically determined manner. Thus the number of linear independent vectors to be provided by the test had to be six also. Normally this can only be accomplished by 6-axis shaking tables, for example, if each axis is excited separately (3 translational axes + 3 rotational axes). Lacking a 6-axis shaking table a different approach had to be applied. The idea was to mount the structure on a elastically supported seismic block and excite this seismic block by hammer impacts at six different locations in order to produce six linear independent vectors.

In order to measure the interface forces a force measurement device (FMD) had been used as an interface between the seismic block and the TSP. The measured forces where related to a known point on the FMD. Thus the TSP structure could be regarded as being mounted in a statically determined way at this very point.

The excitation forces had been applied to the seismic block with a heavy rubber hammer and the frequency range to be examined was specified from zero to 200 Hz. The rubber hammer autospectrum showed that this frequency range could be excited sufficiently.

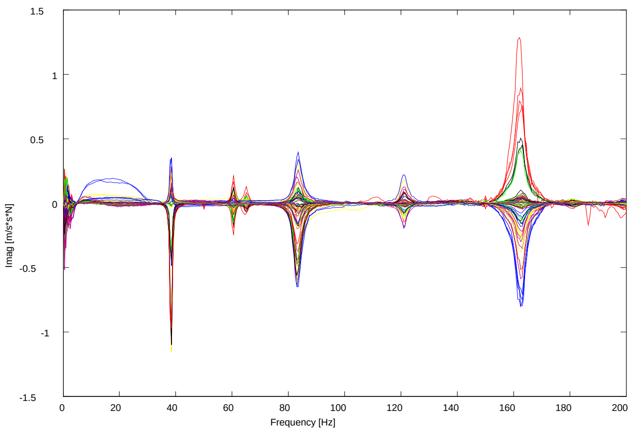
For each of the six defined excitation locations the Fourier transforms of six base accelerations, six interface forces and 65 structural responses (all measurement dof were located on the CFRP plate) have been calculated (15 times for averaging).

Identification of Modal Data

Free/Free Structure.

The first step after acquisition of the test data in form of Fourier transforms was to estimate those six columns of the free/free structure's FRF matrix which are related to the interface forces. This has been done using in-house software that is based on the theory presented above. In figure 3 all 65 estimated FRFs (imaginary parts) with respect to unit interface force in z-direction are shown (third column of the FRF matrix (11)).

Figure 3: Free/free FRFs with respect to unit z-force



The six columns of the FRF matrix served as six independent references that were used for the following modal identification process. Thus theoretically all eigenfrequencies and mode shapes could be identified six times - once for each reference.

Of the overall nine elastic mode shapes of the free/free structure found by the FE analysis in the observed frequency band, only eight have been identified. The mode shapes forming the final result were identified from those references with the highest signal to noise ratio. For example, to identify the mode at 112.61 Hz the unit interface force in y-direction was used since this mode was not excited by the unit z-force according to figure 3. Table 1 shows the eigenfrequencies, modal damping ratios and modal masses that are related to the eight chosen mode shapes. In addition the analytic eigenfrequencies and the MAC values are shown also (MAC values are a means for comparison of two mode shapes. MAC = 0 $\rightarrow$ modes are orthogonal, MAC = 1 $\rightarrow$ modes are collinear).

Table 1: Modal data of free/free TSP structure

#	Freq. (FEA) [Hz]	Freq. (Test) [Hz]	MAC value [-]	Modal Mass [kg]	Modal Damping [%]
1	38.82	38.31	0.94	<u>2.12</u>	0.67
2	61.29	60.50	0.93	0.58	1.02
3	77.51	65.00	0.75	<u>0.58</u>	1.33
4	83.76	83.25	0.92	<u>2.04</u>	1.96
5	85.40	-	-	-	-
6	110.36	112.61	0.98	1.68	1.15
7	161.85	120.95	0.73	<u>0.64</u>	1.05
8	171.03	162.37	0.94	<u>0.50</u>	1.21
9	188.53	181.50	0.99	-	0.96

The underlined modal masses showed the smallest variations during the identification process and are therefore more reliable than the others.

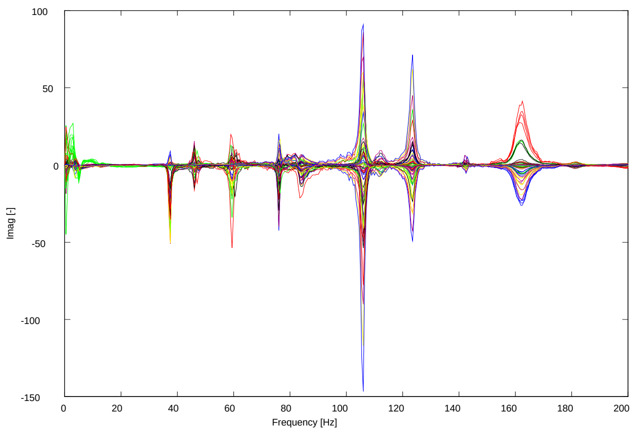
The identified mode shapes were almost free from noise effects and the results show that the estimation of FRFs with respect to measured interface forces of the fixed/free structure provides a basis for the identification of the free/free structure's modal data. Only one free/free mode shape could not be identified from the FRFs because it was not excited significantly.

Fixed/Free Structure.

The first step after acquisition of the test data in form of Fourier transforms here was to estimate those six columns of the fixed/free structure's FRF matrix which are related to the base accelerations. This has been done using in-house software that is based on the theory presented above. In figure 4 all 65 estimated FRFs with respect to unit base accelerations are shown (third column of the FRF matrix (23)).

Comparing the FRFs with respect to unit z-interface force in figure 3 with those with respect to unit z-base accelerations in figures 4 indicates that the latter exhibit more noise. This lack of accuracy lead to uncertainties of the estimated FRFs that may be traced back to the low base acceleration levels resulting from the limited impact force to excite the seismic block (low signal to noise ratio).

Figure 4: Fixed/free FRFs with respect to unit z-base acceleration



The six columns of the resulting FRF matrix again served as six independent references that were used for the following modal identification process.

All analytic mode shapes of the fixed/free structure could be identified from at least two different references although the visual quality of some mode shapes was very poor. The mode shapes forming the final result were taken from those references that yielded the best signal to noise ratio. Table 2 shows the eigenfrequencies, MAC values (Test/FEA) and modal damping ratios that are related to the 12 identified mode shapes. The modal masses could not be identified with standard EMA software because the effective excitation force on a structure under base excitation is unknown. However these values could be identified using another in-house software product that is based on the theory presented by Link and Qian (1994) using the measured interface forces and are also presented in table 2.

Table 2: Modal data of fixed/free TSP structure

#	Freq. (FEA) [Hz]	Freq. (Test) [Hz]	MAC value [-]	Modal Mass [kg]	Modal Damping [%]
1	34.56	37.43	0.96	1.50	0.54
2	47.50	46.25	0.93	1.23	0.65
3	56.72	59.00	0.94	0.30	0.76
4	61.03	60.50	0.89	0.14	0.83
5	75.20	76.00	0.88	7.8	0.35
6	80.05	83.25	0.86	2.54	0.89
7	98.23	106.00	0.97	1.91	0.69
8	110.10	112.50	0.88	-	1.11
9	123.23	123.50	0.97	1.76	0.72
10	136.96	142.00	0.81	10.44	0.63
11	163.79	161.90	0.98	0.66	1.69
12	188.40	181.39	0.91	-	1.35

It can be seen that the MAC values are very good for all mode shapes. Nevertheless due to the observed relatively low signal to noise ratio some results for the modal damping and modal mass values appeared to be sensitive to identification control parameters and should be considered as mean values with considerable scatter.

CONCLUSIONS

The theory and application presented in this paper show that a complete identification of free/free and fixed/free modal data can be accomplished by merely testing the fixed/free system with an additional measurement of the reaction forces if n_b linear independent base excitation vectors are provided.

I.e. if the interface forces and displacements (accelerations) of a fixed/free system are measured in addition to the structural responses of the system in an appropriate manner ($\rightarrow n_b$ linear independent base excitation axes) one has doubled the knowledge of the system's eigenbehavior. Thus, if we are able to measure interface forces we can cut down testing time and testing costs which represents an obvious advantage of the methods described above.

However the application has also shown that a sufficient base excitation level has to be supplied in order to achieve reliable estimation and identification results.

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