# COMPUTATIONAL MODEL UPDATING AND VALIDATION OF AERO-ENGINE FINITE ELEMENT MODELS BASED ON VIBRATION TEST DATA

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Abstract. In this paper, an overview and some selected details are given on a general procedure to improve large order structural dynamic models of aero-engines. Because of the complexity of typical whole engine models (WEM) a special model validation strategy is required to account for the large number of finite element (FE) model parameters which may be uncertain in a WEM. A three-step strategy is proposed which is based on three subsequent levels of model validation. In the first step, model validation is carried out on component model level, whereas in the second step, sub-assembly models are validated with emphasis on updating joint parameters, and in the final step, model validation is carried out on the whole engine assembly level where only a few uncertain joint parameters need to be adjusted. Typical problems encountered during computational model updating are addressed, among those, the selection of effective updating parameters. An extension of the classical model updating techniques is presented which aims at extending the prediction capability of the model into the large vibration amplitude regime where non-linear behavior is likely to occur. This is achieved by identifying non-linear joint stiffness and damping parameters by application of a non-linear model updating approach based on measured non-linear frequency response data.

### **1 INTRODUCTION**

Finite element models are extensively used in aero-engine product design cycles to predict the response of the engine when exposed to certain loading conditions. The intended use of such FE models is to gradually replace expensive tests on prototypes and/or to improve the test design using FE simulations.

The accuracy of the response predictions obtained with FE simulations is largely dependent on the FE model quality, which itself is mainly governed by the assumptions made during model generation. Once an FE model was generated (which shall be called the initial model), it should be subjected to a quality check to assess the accuracy of the prediction results. Usually, experimental modal data as well as inertia properties (e.g. rigid body mass matrix) are used for this purpose. If the quality check reveals that discrepancies between model predictions and experimental data are due to erroneous assumptions made for certain model parameters (instead of model structure errors), then computational parameter updating can be applied to improve the initial prediction capability of the model by means of adjusting simultaneously a number of model parameters which are considered to be uncertain.

It is well known that the excitation force levels applied during modal survey testing are far away from the excitation force levels which a structure may face under operating conditions. Furthermore, many structures exhibit different dynamic behavior under large excitation force levels (i.e. large vibration amplitudes) due to non-linearities which are an inherent property of most real structures. Hence, validating a model using the underlying linear modal data may not always be sufficient to finally obtain a predictive model. Assuming that the behavior of the structure in the low vibration amplitude regime (i.e. low excitation force levels) can be described with sufficient accuracy by a linear model, then the experimental modal data can only be used to validate the so called underlying linear model. In addition, carefully designed vibration tests using large excitation force levels can be used to characterize the non-linear behavior of the structure. The basic idea followed in this paper is that after the underlying linear model has been validated, this model can be extended to account for local non-linearities like those occurring at joints. Afterwards, measured non-linear response data can be used in a computational model updating procedure to identify/improve the parameters of the non-linear elements introduced at the joints to account for the non-linear behavior. This extension of the classical computational model updating approach and was developed and applied to a whole aero-engine model within the European research project CERES.

### 2 MODEL AND TEST DATA QUALITY ASSESSMENT

In [1] it is stated that model errors mainly arise from three different sources. These are idealization errors, discretization errors, and erroneous assumptions for model parameters.

Idealization errors result from the assumptions made to characterize the mechanical behavior of the structure. This involves the selection of element types and element formulations used to represent certain model parts, for instance, if beam elements are appropriate to model spokes and vanes of aero-engine casings, or if other element types are better suited.

Discretization errors are introduced by numerical methods inherent in the finite element method. Among those are typical meshing problems, for example, if a certain mesh of finite elements with their inherent shape functions is appropriate to approximate the true deformation of a structure under a given loading condition, or if a mesh refinement is necessary.

Erroneous assumptions for model parameters are introduced wherever the properties (stiffness, mass, and damping) of elements can only be roughly estimated such that the level of confidence in these properties is relatively low. Examples are modeling shell areas of varying thickness by using a mesh of shell elements with constant thickness, or the determination of the stiffness and damping properties of a simple bolted joint.

Idealization and discretization errors **cannot** be corrected automatically and have to be detected by engineering judgment during an initial model quality check. Erroneous assumptions for model parameters can be corrected by using computational model updating. However, this will only yield reasonable results if the initial model structure is physically meaningful. From that point of view, computational model updating can be considered as a simultaneous fine tuning of a number of model parameters which cannot be done manually within reasonable time scales.

The requirements posed on the quality of FE models w.r.t. their final utilization are discussed in [1] and are given there in terms of acceptable frequency deviations and acceptable MAC values [2] between experimental modal data and analytical modal data obtained with the FE model. These requirements are pretty much dependent on the type of structure under consideration and the quality of the test data which is available so that absolute numerical limits for frequency deviations and MAC values can generally not be specified.

Another aspect to be considered is the quality and the quantity of the test data. In many cases the FE engineer who is responsible for the FE model is supplied with experimental modal data without information about the quality of the experimental modal analysis. However, this information is necessary to check if the quality of the experimental modal data is sufficiently high that it is justified to use it for model updating. Test data quality assessment would enclose the test setup, the frequency response functions, and the extracted modal data. Items like quality of excitation, sensor location, repeatability, reciprocity, and robustness of extracted modes with respect to different extraction methods should be assessed [3], [4].

#### **3 OUTLINE OF UPDATING THEORY**

Computational model updating is based on the parameterization of the system matrices of the model according to the so called substructure matrix approach [5, 6, 7]. However, in order to generalize the updating theory to the non-linear case some extensions have to be introduced. In general, the parameterization of a model is done by a Taylor series expansion of the system matrices of the FE model:

$$[M] = [M_A] + \sum_{i=1}^{I} m_i \cdot \frac{\partial [M]}{\partial m_i} \cdot \alpha_i , \qquad (1a)$$

$$[K] = [K_A] + \sum_{j=1}^{J} k_j \cdot \frac{\partial [K]}{\partial k_j} \cdot \beta_j, \qquad (1b)$$

$$[C] = [C_A] + \sum_{k=1}^{K} c_k \cdot \frac{\partial [C]}{\partial c_k} \cdot \gamma_k .$$
(1c)

In equations (1),  $[M_A]$ ,  $[K_A]$ ,  $[C_A]$  are the mass-, stiffness-, and damping matrix of the initial model,  $m_i$ ,  $k_j$ ,  $c_k$  are the actual values for the uncertain mass-, stiffness-, and damping parameters, and  $\alpha_i$ ,  $\beta_j$ ,  $\gamma_k$  are the dimensionless design parameters to be updated (updating parameters). If the erroneous mass-, stiffness-, and damping parameters  $m_i$ ,  $k_j$ ,  $c_k$  appear linear in the overall system matrices, then the well known substructure matrix approach for FE model parameterization is obtained [5, 6, 7] with constant substructure mass-, stiffness-, and damping matrices defining type and location of the model error:

$$\left[\boldsymbol{M}_{i}\right] = \boldsymbol{m}_{i} \frac{\partial \left[\boldsymbol{M}\right]}{\partial \boldsymbol{m}_{i}}, \qquad (2a)$$

$$\left[K_{j}\right] = k_{j} \frac{\partial \left[K\right]}{\partial k_{j}},$$
(2b)

$$\begin{bmatrix} C_k \end{bmatrix} = c_k \frac{\partial \begin{bmatrix} C_k \end{bmatrix}}{\partial c_k}.$$
 (2c)

These substructure matrices may either affect a single element, a group of elements (e.g. all elements which refer to the same material property), or even a complete component model of an overall assembled model. This means that a single design parameter  $\alpha_i$ ,  $\beta_j$ ,  $\gamma_k$  may affect a certain portion of the overall stiffness-, mass-, or damping matrix and thus determines type and location of the assumed model error.

The unknown design parameters which describe the impact of the model errors related to  $[M_i], [K_j], [C_k]$  are comprised in the parameter vector  $\{p\}$ :

$$\{p\} = \{\alpha_1 \quad \cdots \quad \alpha_I \mid \beta_1 \quad \cdots \quad \beta_J \mid \gamma_1 \quad \cdots \quad \gamma_K\}^I, \qquad (3a)$$

$$\{p\} = \{p_1 \quad \cdots \quad p_{np}\}^T, np = I + J + K.$$
 (3b)

The parameterization of the FE model according to equations (1) allows for local updating of uncertain model areas. Using these equations together with appropriate residuals containing the test/analysis deviations the following objective function J can be derived:

$$J(\lbrace p \rbrace) = \lbrace r(\lbrace p \rbrace) \rbrace^{T} [W] \lbrace r(\lbrace p \rbrace) \rbrace \to \min.$$
<sup>(4)</sup>

In this equation,  $\{r(\{p\})\}\$  is the residual vector which is a function of the unknown design parameters and [W] is a symmetric weighting matrix. The objective function (4) contains the weighted square sum of the test/analysis differences and shall be minimized during updating. The residual vector  $\{r(\{p\})\}\$  holds the difference between test data comprised in vector  $\{u^t\}\$  and the analytical data comprised in vector  $\{u^a(\{p\})\}\$ :

$$\left\{r\left(\left\{p\right\}\right)\right\} = \left\{u^{t}\right\} - \left\{u^{a}\left(\left\{p\right\}\right)\right\}.$$
(5)

Since the analytical data vector  $\{u^a(\{p\})\}\$  usually depends in a non-linear way on the design parameters, the minimization problem of equation (4) is also non-linear and must be solved iteratively. One way to solve such a non-linear minimization problem is the classical sensitivity approach, where the analytical data vector is linearized at point 0 by a Taylor series expansion truncated after the first term:

$$\left\{u^{a}\left(\left\{p\right\}\right)\right\} = \left\{u_{0}^{a}\right\} + \left[G_{0}\right]\left\{\Delta p\right\}.$$
(6)

residual vector at the linearization point 0

Introducing the Taylor series expansion of the analytical data vector into equation (5) leads to:

$$\{r(\{p\})\} = \{u^t\} - \{u_0^a\} - [G_0]\{\Delta p\} = \{r_0\} - [G_0]\{\Delta p\},$$
(7)

with:

 $\{r_0\} = \{u^t\} - \{u_0^a\}$ 

$$\begin{bmatrix} G_0 \end{bmatrix} = \frac{\partial \left\{ u^a \left( \left\{ p \right\} \right) \right\}}{\partial \left\{ p \right\}} \bigg|_{\left\{ p \right\} = \left\{ p_0 \right\}}$$
 sensitivity matrix at the linearization point 0  
$$\{\Delta p\} = \left\{ p \right\} - \left\{ p_0 \right\}$$
 design parameter changes  
$$\{ p_0 \}$$
 design parameters at linearization point 0

Introducing the linearized residual (7) into the objective function (4) and introducing additionally a so-called regularization term to account for ill-conditioned sensitivity matrices yields the quadratic objective function:

$$J\left(\{\Delta p\}\right) = \underbrace{\left\{r\left(\{\Delta p\}\right)\right\}^{T}\left[W\right]\left\{r\left(\{\Delta p\}\right)\right\}}_{residual\ term} + \underbrace{\left\{\Delta p\right\}^{T}\left[W_{p}\right]\left\{\Delta p\right\}}_{regularization\ term},$$
(8)

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with the symmetric weighting matrix  $\begin{bmatrix} W_p \end{bmatrix}$  (regularization matrix) to constrain excessive parameter variations in each iteration step.

If the design parameters are not bounded the minimization of the objective function (8)

$$J(\{\Delta p\}) \to \min \quad \Rightarrow \quad \frac{\partial J(\{\Delta p\})}{\partial \{\Delta p\}} = \{0\}$$
(9)

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leads to the following equation for the calculation of design parameter changes at the linearization point 0:

$$\{\Delta p\} = \left[ \left[ G_0 \right]^T \left[ W \right] \left[ G_0 \right] + \left[ W_p \right] \right]^{-1} \left[ G_0 \right]^T \left[ W \right] \{r_0\}.$$
<sup>(10)</sup>

The parameter changes calculated from equation (10) are added to the initial parameter values at the linearization point and the objective function must be evaluated again using the new design parameters  $\{p\}$ :

$$\{p\} = \{p_0\} + \{\Delta p\}.$$
(11)

The procedure of calculating a vector of design parameter changes  $\{\Delta p\}$ , adding them to a given vector of initial design parameters  $\{p_0\}$ , and the computation of the results of the structure with the new vector of design parameters  $\{p\}$  continues until the objective function (4) drops below a certain threshold value, or respectively, until the test/analysis deviations converge to an acceptable level w.r.t. the intended usage of the model [1].

The residuals used in computational model updating can be manifold and many of them have been proposed. Most frequently, eigenfrequency and mode shape deviations are utilized. In this case, the residual vector at the linearization point 0 is made up of eigenfrequency and mode shape differences (see [5, 6, 7]). In case of models with non-linear parameters, the theory of computational model updating remains unchanged, however, the type of residual has to be adopted to the specific problem at hand. In the non-linear updating example to be presented below the residual vector contains differences between experimental and analytical frequency response functions (FRFs) calculated and measured at different load levels. These FRFs are typically measured for a limited number of discrete frequencies  $\Omega_i$  due to the application of signal processing procedures during the measurements. Thus, the residual vector in case of non-linear updating has the following structure:

$$\{r_{0}\} = \begin{cases} H_{jk}^{t}(\Omega_{1}) - H_{jk}^{a}(\Omega_{1}) \\ \vdots \\ H_{jk}^{t}(\Omega_{n}) - H_{jk}^{a}(\Omega_{n}) \end{cases} \right|_{\{p\} = \{p_{0}\}} ,$$
(12)

with  $H_{jk}^{i}(\Omega_{i})$  being the experimental FRF at frequency point  $\Omega_{i}$  measured at the degrees of freedom (DoF) j = 1, ..., n when the excitation force was applied at DoF k, and  $H_{jk}^{a}(\Omega_{i})$  is the corresponding analytical FRF obtained at the linearization point 0. The sensitivity matrix for the frequency response residual vector contains the derivatives of the analytical FRFs w.r.t. the design parameters:

$$\begin{bmatrix} G_0 \end{bmatrix} = \begin{bmatrix} \frac{\partial H_{jk}^a(\Omega_1)}{\partial p_1} & \cdots & \frac{\partial H_{jk}^a(\Omega_1)}{\partial p_{np}} \\ \vdots & \ddots & \vdots \\ \frac{\partial H_{jk}^a(\Omega_n)}{\partial p_1} & \cdots & \frac{\partial H_{jk}^a(\Omega_n)}{\partial p_{np}} \end{bmatrix}_{\{p\}=\{p_0\}}$$
(13)

Currently, these derivatives are approximated by a finite difference approach within the nonlinear updating software Update\_NL. In case of non-linear updating the FRFs for different levels of excitation force are stacked in the residual vector (12) and in the sensitivity matrix (13).

#### 4 COMPUTATIONAL MODEL UPDATING OF AN AERO-ENGINE WEM

The application of computational model updating to large order FE models often is accompanied by two basic problems. The first problem is the large number of FE model parameters which must be considered uncertain. This leads to a large optimization problem which may not be solved uniquely, i.e. a number of possible parameter combinations may exist which all fulfill the minimum criterion for the objective function. In such cases, the calculated parameter changes will suffer from the errors of ill-conditioning of the equation system increasing with the number of parameters and the result of computational model updating may be highly sensitive to the number of updating parameters and to the content of the residual vector. The second problem of updating large order FE models is related to the limited amount of test data available for updating. There is a rule of thumb which states that the number of updating parameters shall not exceed the number of eigenfrequencies available from a test. From these two problems of updating large order FE models it becomes clear that a strategy for updating is required to make updating of large order FE models feasible at all.

The updating strategy which is followed here was already applied in [8] and makes use of disassembling the overall structure into its components and to update the underlying linear models of these components individually using experimental modal data obtained from component tests with either unconstrained boundary conditions or with mass loaded boundaries. Afterwards, subsets of the updated components are assembled to form subassemblies, mainly in order to update the interface parameters like joint stiffnesses introduced between the components. Of course, at this stage it may also be necessary to update other parameters as well, for example, stiffness parameters which appeared to be insensitive in component model updating.

After subassembly models have been updated, the whole engine assembly model is formed from the updated component models and the updated subassembly models. At this stage a model correlation with whole engine test data from low level modal survey testing should already yield reasonable results, so that only minor adjustments are necessary of some interface parameters or of some other model parameters which appeared to be insensitive in the previous updating stages.

When this last stage of linear computational model updating has been performed successfully one can proceed to model non-linear effects at the joints to account for non-linear behavior in the large amplitude regime. It should be noted that an underlying linear model validated in the previous updating stages is a prerequisite for non-linear modeling and updating. This linear and subsequent non-linear modeling and updating strategy was applied to the aero-engine shown in figure 1 within the European research project CERES in which four universities and three aero-engine manufacturers worked together to improve the prediction capabilities of whole engine FE models.

The computational model updating software ICS.sysval was utilized for linear model updating using modal data, whereas University of Kassel's updating software Update\_NL was utilized for non-linear updating using frequency response data where the non-linear frequency response software HBResp was used to compute the non-linear analytical frequency response functions.



Figure 1: Aero-engine with accessories and WEM w/o accessories (ca. 105.000 degrees of freedom)

## 4.1 Computational model updating of a single component

In this chapter some details are presented on updating a single component to explain the basic procedure of selecting meaningful updating parameters. Computational model updating of the CCOC (combustion chamber outer casing, component 5 in figure 7) shall be discussed here as a typical example for updating of single components.

In principle, the CCOC (see figure 2) is a thin walled cylindrical shell with flanges attached to the ends of the cylinder. The height of these flanges is varying in circumferential direction (so called scalloped flanges). As can be seen from figure 2, the structure has lots of bosses, holes and outtakes, which, however, do not disturb the rotational symmetry significantly. Some of these locally thickened shell areas also appear in a cyclic symmetrical pattern which supports the high degree of axisymmetry. The FE model was generated using the FE code MSC.Nastran, hence, some of the MSC.Nastran specific vocabulary is used in the following for the description of the updating parameters.



Figure 2: CCOC with tuning masses attached to the flanges for modal survey test

Computational model updating of the linear CCOC component model started with correlating analytical and experimental modal data. The experimental data was obtained from a modal survey test with the CCOC suspended by soft cords to simulate free/free conditions (see figure 2). Tuning masses were attached to front and rear flanges during the tests to enforce a certain spatial orientation of the ovalizing shell modes and thereby to make the comparison of the analytical and experimental mode shapes easier (these masses are also part of the FE model and were removed after component model updating).

The experimental modal data available for updating consists of 27 modes in the frequency range up to 1300 Hz. A correlation of experimental data and analytical data of the initial

model revealed that the model was much too stiff such that remodeling was necessary (only 10 out of 27 modes could be paired with MACs higher than 50%, the mean absolute frequency deviation of the paired modes was about 19.5% and the average MAC of the paired modes was 77.4%). It was found that circumferential rings of beam elements were the main reason for the initial model being too stiff. The intended purpose of these beam rings was to fit the FE model geometry to the CAD geometry of the real structure especially in areas with transitions in shell thicknesses and in areas with fillet radii (e.g. where flanges are connected to the casing shells). The stiffening effect of these beam rings was much more pronounced than it was expected so that they were either removed completely, or their connectivity was changed in such a way that one beam ring was no longer attached to only one circumferential ring of nodes but rather to more than one ring of nodes. Also some minor corrections related to the attachment of the tuning masses and their inertia properties had to be introduced. This kind of remodeling affects the mathematical structure of the model and cannot be updated automatically by numerical procedures. A subsequent correlation of the model results with experimental data was performed and led to much better results, see figure 3 (20 out of 27 modes could be paired with MACs higher than 50%, the mean absolute frequency deviation of the paired modes was about 8.1% and the average MAC of the paired modes was 80.5%).



Figure 3: Correlation results of test data and analytical data of the initial model after remodeling

Before starting computational model updating, a sensitivity analysis was performed for 32 model parameters (MSC.Nastran property card entries) which were assumed to be uncertain and which were selected by engineering judgment. Among them were, for example, the shell thicknesses of 15 circumferential rings of shell elements (T entry of the PSHELL card), the cross sectional properties of stiffening beam rings (A, I1, I2, J entries of the PBAR card), and widths and heights of the scalloped flanges which were either modeled by circumferential beam rings or by rings of shell elements. In case of a beam representation of a flange the cross sectional properties as well as the beam offset (W1A, W1B, W3A, W3B entries of the CBAR card) were considered as parameters, whereas in case of a shell representation of a flange the shell element thickness and the radial coordinate of the element nodes were considered as parameters.

Three sets of possible updating parameters were selected out of the 32 candidate parameters. The first set was assembled from the 10 most sensitive parameters in terms of eigenfrequency sensitivity w.r.t. only those eigenfrequencies which showed large deviations in the initial correlation. The second set was assembled from the 10 most sensitive parameters in terms of mode shape sensitivity w.r.t. only those mode shapes which had low MACs in the initial correlation. The third set was assembled from the 10 most effective parameters for reducing the so called equation error, see [9]. A final set of updating parameters was then selected from

the three different sets by finding those parameters which are members in all three independent sets or in at least two of these sets.

The 10 most effective model parameters were selected for updating from the three different sets, but it was found out that these 10 parameters were not completely independent like, for example, the cross sectional area and the area moment of inertia of a beam. By using nonlinear relations among the 10 selected parameters it was possible to further reduce the number of updating parameters to only four independent parameters, which, however, did not only affect the 10 model properties considered for updating, but also 6 other model properties which were not considered previously. Table 1 comprises the four independent updating parameters and their relation to 16 different model properties (i.e. MSC.Nastran property card entries). Each of the 16 Nastran property card entries is related in a linear or non-linear way either to a single or to multiple independent updating parameters. These non-linear parameterto-property relations can be derived by basic engineering equations such as the one shown in equation (14), where it can be seen that the area moment of inertia  $I_1$  of a beam with rectangular cross section and rigid offset is non-linearly dependent on three independent parameters, namely the beam height h, the beam width b, and the thickness t of the shell element to which the offset beam is attached (see figure 4 for the connectivity of the beam element and the shell element).

$$I_1(b,h,t) = \frac{bh^3}{12} + hb\left(\frac{h+t}{2}\right)^2$$
(14)



Figure 4: Beam element eccentrically connected to a shell element node by a rigid offset

Such a non-linear parameter-to-property relation can be defined by MSC.Nastran DVxREL2 cards in conjunction with DEQUATN cards and are used in SOL 200 (design sensitivity and optimization) utilized by the updating software ICS.sysval.

Table 1 shows that not only stiffness properties are affected by the four updating parameters, but also so called non-structural mass properties (NSM entry of the PSHELL card). The relation to these non-structural mass entries were introduced in order to keep the overall FE model mass constant during model updating, because the material density was already adjusted to meet the weighted mass of the real structure.

It should be noted here that the parameters were selected based on a sensitivity analysis and on engineering judgment. A sensitivity analysis, however, only indicates whether parameters are sensitive but does not give an indication about the error on the parameters. For example, there may be parameters in the FE model which are definitely wrong but have not been selected as updating parameters because of their low sensitivities. Insensitive parameters should not be used as updating parameters, because in this case it must be expected that these parameters cannot be identified from the test data. Insensitive parameters would also suffer from unreasonably high parameter changes during computational updating which cannot be explained by physical reasoning but from the behavior of mathematical optimization. Parameters which are insensitive in case of single component model updating may become more sensitive when they are contained in a subassembly model or even in the whole engine model. If some parameters are still insensitive there, it can be concluded that they cannot be identified from the available test data.

Parameter Number	Parameter Description	Parameter to Nastran Property Card Relation	Property Card Entry
1	height of scalloped front flange	beam cross section properties of front flange (PBAR)	A  1  2 J
		beam offset of front flange (CBAR)	W1A W1B
2	width of scalloped front flange	beam cross section properties of front flange (PBAR)	A  1  2 J
		beam offset of front flange (CBAR)	W3A W3B
3	thickness of 3 neighboring rings of shell elements	shell thickness and non-structural mass (PSHELL)	T NSM
		shell thickness and non-structural mass (PSHELL)	T NSM
		shell thickness and non-structural mass (PSHELL)	T NSM
		beam offset of rear flange affected by shell thickness change (CBAR)	W1A W1B
4	thickness of conical shell elements	shell thickness and non-structural mass (PSHELL)	T NSM

Table 1: Final set of updating parameters and their relation to other model properties

Figure 5 shows the evolution of the eigenfrequency errors, the MAC values, and the updating parameters during 16 updating loops. It can be observed that the parameters already converge after 8 iterations.



Figure 5: Track plots of updating the CCOC model parameters

The correlation after update (see figure 6) showed that 21 out of 27 modes could be paired with MACs higher than 50%, the mean absolute frequency deviation of the paired modes was about 4.4% and the average MAC of the paired modes was 82%. The last six modes in figure 6 represent the so called passive frequency range, i.e. these modes were not considered for updating (were not contained in the residual) but are used to check the prediction capability of the model. In this case, almost no improvement of the frequency deviations of the passive modes could be achieved. Since the influence of the passive modes of the CCOC model on the whole engine dynamics was found to be negligible no further attempts were made to improve the CCOC model in that frequency range.



Figure 6: MAC matrix and frequency deviations after update

The CCOC updating example shows how meaningful updating parameters can be selected based on different sensitivity studies and how only a few sensitive parameters can affect a relatively large number of FE model properties by defining non-linear relations among them. A common problem frequently encountered in updating of aero-engine components is the axisymmetry of some components. This axisymmetry property of the structure may lead to mode pairing problems since experimental and analytical mode shapes may appear rotated w.r.t. each other. Poor MAC values and cross pairing of modes which are close in frequency (even though the overall frequency deviations are acceptable) is one indicator for the problem of rotated modes. Different approaches for updating of axisymmetric structures are presented in [10] but shall not be discussed here.

#### 4.2 Computational model updating of sub-assembly models

The idea of updating subassembly models was introduced since the whole engine model had too many interfaces (7 interfaces between 8 components in case of the aero-engine shown in figure 7). All these interfaces were modeled by flexible springs according to figure 8. The joints were idealized as flexible hinges at the bolt lines allowing for an axial opening of the joint bases. The axial opening was constrained by springs, and the corresponding spring stiffnesses served as potential updating parameters. In addition, it turned out that the joint stiffnesses are most sensitive w.r.t. the global bending modes of the whole engine structure, whereas the sensitivity w.r.t. most ovalizing shell modes turned out to be relatively low. This means that a relatively large number of joint stiffnesses had to be updated from a limited number of modes. It is believed that such an updating attempt is prone to ill-conditioning and is expected to yield unreasonable parameter changes. In order to improve the conditioning for updating the joint stiffnesses it was decided to reduce the number of joint stiffness parameters to be updated simultaneously. This can be achieved by generating subassembly models from assembling different component models validated in the previous updating stage. Figure 7

shows the whole engine model and five different subassemblies where some subassemblies contain only one isolated joint and other subassembly models have multiple joints. There is also a bit of redundancy, because some joints are part of multiple subassembly models. All these subassemblies had to be tested individually and in some cases large masses had to be attached to the free interfaces in order to activate the joint stiffnesses in the tests. The modal data of these different subassemblies provided the experimental database for joint stiffness updating.



Figure 7: Exploded view on whole engine model and different subassembly models

In principle, less effort is involved in updating subassembly models compared to updating single component models. In component model updating the selection of meaningful updating parameters is the major step and can be very time consuming. In subassembly updating, where the subassemblies were generated from validated component models, it is clear from the beginning which parameters have to be updated. Therefore, in subassembly updating it has to be checked if all joint stiffness parameters can be updated simultaneously, or if some joint stiffnesses are relatively insensitive and should therefore not be considered as updating parameters. If this was checked, updating proceeds in exactly the same way as component model updating which has already been demonstrated on the CCOC example.



Figure 8: Principle arrangement for joint modeling

### 4.3 Whole engine computational model updating of non-linear parameters

Non-linear updating of the whole engine model outlined here to point out the updating strategy of the different stages of linear and non-linear computational model updating. Details of the non-linear updating work can be found in references [11, 12].

The non-linear updating software Update\_NL utilizes non-linear FRFs for updating non-linear parameters, e.g. those representing the joint non-linearities. A test setup with the whole engine supported by bungee cords was chosen for the non-linear tests. Step-sine tests with different constant excitation force levels ranging from 1N (quasi-linear) up to 70N excitation force were performed to measure a set of non-linear FRFs. Figure 9 shows the test setup with the bypass duct (component 8 in figure 7) removed from the whole engine assembly. This was done because of the large number of ovalizing shell modes of this component which are usually suppressed by the real boundary conditions of the structure. The measurement DoFs at which the non-linear FRFs were measured are shown schematically on the FE model (ca. 90.000 DoFs) in figure 9.



Figure 9: Test setup for non-linear step-sine tests and FE model with measurement positions

It was expected that the fundamental bending mode of the whole engine assembly shows nonlinear behavior in the large vibration amplitude regime. Hence, the step-sine tests were performed in a narrow frequency band around the resonance frequency of this bending mode. The experimental non-linear FRFs measured at the driving point are shown in figure 10 for different excitation force levels where it can be observed that the resonance frequency shifts towards lower frequencies and that the damping decreases with increasing excitation force levels.



Figure 10: Non-linear experimental FRFs measured at the driving point

From FE simulations it was found out that only the joint between the intermediate casing (IMC, component 4 in figure 7) and the CCOC marked in figure 9 was sufficiently loaded (i.e. was reasonably sensitive) in the narrow frequency band investigated in the tests. Thus, it was decided only to model this joint in a non-linear way by supplementing the springs at the joint base shown in figure 8 by appropriate non-linear elements. The softening stiffness non-linearity of the joint was modeled by 48 bilinear springs equally spaced on the circumference between the IMC and the CCOC, whereas the non-linear damping characteristic was modeled by 48 softening quadratic dampers (in addition to the underlying linear modal damping). The non-linear restoring forces of these elements are shown in figure 11.



Figure 11: Restoring force of bilinear spring and softening quadratic damper

The compression regime stiffness  $k_1$  of the bilinear spring represents the underlying linear joint stiffness and was identified using linear modal data (either from linear subassembly updating, or from linear whole engine updating). Therefore, only three non-linear parameters must be updated. These are the tension regime stiffness  $k_2$  and the transition point  $u_c$  of the bilinear spring and the damper constant  $c_{nl}$  of the softening quadratic damper. The underlying linear stiffness  $k_1$  is left unchanged which preserves the underlying linear model results in the low amplitude regime.

The residual vector was assembled from the non-linear FRFs measured at DoFs 1 to 4 of two different load levels, 20 N and 70 N excitation force (i.e. FRFs from the highest and from a moderate load level). Figure 12 shows the evolution of the updating parameter changes and of the normalized objective function during 10 updating iterations.



Figure 12: Evolution of non-linear updating parameters and normalized objective function

After non-linear updating the prediction capability of the model in the large amplitude regime was checked by comparing the analytical and experimental non-linear FRFs of those excitation force levels which had not been used for updating. Such a comparison is shown in figure 13 for the drive point measurement. It can be observed that after updating the non-linear parameters the predicted non-linear FRFs are close to measured ones. Similar results as those shown in figure 13 were obtained at the other measurement DoFs as well but are not shown here (see references [11, 12]).



Figure 13: Comparison of analytical and experimental non-linear FRFs for different excitation force levels after non-linear updating

#### **5 SUMMARY**

In computational model updating, selected FE model parameters are adjusted in such a way that the deviations between FE model predictions and experimental results are minimized. Computational model updating represents a partly automated procedure for the validation of FE models which yields satisfactory results if the initial model and the test data fulfill a number of quality requirements.

If the updating parameters really represent the true and only sources of FE model errors, then computational model updating will yield excellent results. This, however, is a strong requirement which is difficult to achieve in engineering practice and is often the reason for unsatisfactory results of computational model updating when the updating parameters have been selected carelessly. If engineering judgment has been used not only in the selection of parameters but also in the definition of dependencies among them in order to reduce the overall number of independent parameters, then computational model updating can yield satisfactory results even in cases where the selected updating parameters do not represent the true and only source of model error. The resulting model after computational model updating must then be considered as an equivalent model which can be used for certain applications only but which cannot serve as a general purpose model.

The extension of the model validity into the non-linear regime by identifying non-linear joint parameters from experimental non-linear FRFs is a promising technique but is only meaningful when the underlying linear model was already successfully validated. It does not make sense to predict shifts of resonance frequencies due to joint non-linearities which appear to be smaller than the eigenfrequency deviations of the underlying linear model. The identification of non-linear parameters by using frequency responses residuals requires that the predicted non-linear responses are not too far away from the measured non-linear responses. This would be violated in case of large eigenfrequency errors of the underlying linear model and non-linear updating is then expected to yield unsatisfactory results.

Once the non-linear joint parameters have been identified using non-linear FRFs they can be used for non-linear transient analysis as well. It should be noted, however, that if the response levels which were used for non-linear FRF updating are too far away from the response levels that a structure might face under critical operating conditions, the predictions in the large amplitude regime may become inaccurate, because the non-linear effects may be different at strongly different load levels. This means that accurate predictions using a non-linear model as proposed here requires FRF measurements at relatively high vibration levels which are not always feasible in practice.

#### 6 ACKNOWLEDGEMENT



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